Econ 525a (first half)
Fall 2012
Yale University
Prof. Tony Smith

PROBLEM SET #2

Answers to this problem set are due by the beginning of lecture on Wednesday, September 26. You should submit copies of your code along with a brief description, perhaps in the form of graphs or tables, of your findings. Please submit this documentation by email to: `tony.smith@yale.edu`.

1. Use bisection, the method of successive approximations, the secant method, and Newton's method to compute an estimate of p^*, where $g(p^*) = 0.75$ (and g is defined in the second problem of Problem Set #1). For each method, report how many iterations are required to compute an estimate $\hat{p}$ satisfying $|g(\hat{p}) - g(p^*)| < 10^{-6}$.

2. Repeat the second problem using Brent's method as described in Chapter 9.3 of *Numerical Recipes*. (Feel free to translate the Fortran code in *Numerical Recipes* into a language of your choosing.)

3. Imagine a consumer who either has a low wage ($w = w_1$) or a high wage ($w = w_2$) and whose asset holdings a are restricted to lie in the set $\{a_1, a_2, a_3\}$, where $a_1 < a_2 < a_3$. The wage w follows a discrete-state Markov chain with transition probabilities $\pi_{ij} = P(w' = w_j | w = w_i)$. Let the consumer's savings decision rule $a' = g(a, w)$ take the following form:

   (i) $a_1 = g(a_1, w_1)$, $a_1 = g(a_2, w_1)$, $a_2 = g(a_3, w_1)$

   (ii) $a_2 = g(a_1, w_2)$, $a_3 = g(a_2, w_2)$, $a_3 = g(a_3, w_2)$

   (a) Suppose that $\pi_{22} = 0.95$ and $\pi_{11} = 0.5$. Find the invariant distribution over the two wage levels w_1 and w_2. That is, find two numbers μ_1 and μ_2 such that if fraction μ_i of consumers have wage equal to w_i today, then these fractions replicate themselves in the next period.

   (b) Now find the invariant distribution μ_{ij} over the discrete state space $\{a_1, a_2, a_3\} \times \{w_1, w_2\}$.

   (c) Suppose that consumers are distributed initially according to the invariant distribution that you computed in part (b). What is the probability that a (randomly chosen) consumer with the lowest level of asset holdings today has the highest level of asset holdings three periods from now?

(d) Suppose that initially consumers are spread uniformly over the state space. Compute the dynamics of the distribution of consumers as time evolves and verify numerically that this distribution converges to the one that you computed in part (b). (You may want to write a program to automate the numerical calculations.)