Econ 525a (first half)
Fall 2012
Yale University
Prof. Tony Smith

PROBLEM SET #3

Answers to this problem set are due by the beginning of lecture on Wednesday, October 3. You should submit copies of your code along with a brief description, perhaps in the form of graphs or tables, of your findings. Please submit this documentation by email to: `tony.smith@yale.edu`.

1. Write a program (in a language of your choosing) that uses golden-section search to find the maximum of the function $f(x) = \log(x) - x$.

2. Write a program that uses the Newton-Raphson method to find the maximum of the function in the first problem. Which method converges more quickly?

3. Consider an exchange economy with two periods and two (price-taking) consumers. Consumer i is endowed with ω^i consumption goods in period 1. Consumption goods are not storable. There are two possible states of the world, state 1 and state 2, in period 2, with associated probabilities π_1 and $\pi_2 = 1 - \pi_1$. Consumer i is endowed with ϵ^i_j consumption goods in state j in period 2. Each consumer maximizes

$$U(c^i_1) + \beta E[U(c^i_2)],$$

where c^i_1 is consumer i's period-1 consumption and c^i_2 is consumer i's period-2 consumption (which may depend on the state of the world in period 2). Let $U(c) = (1-\gamma)^{-1}(c^{1-\gamma} - 1)$, where $\gamma > 0$. Finally, assume that the two consumers face symmetric risks in period 2: in particular, set $\pi_1 = \pi_2 = 1/2$ and set ϵ^i_j equal to $2 + z$ if $i = j$ and equal to $2 - z$ if $i \neq j$, where $0 \leq z < 2$. (Note: In cases where you cannot find an analytical answer to the questions posed below you may instead present numerical results.)

   (a) Suppose that markets are complete: in period 1, the two consumers trade two Arrow securities, one for each state of the world in period 2. Let a^i_j be the quantity of Arrow securities purchased by consumer i in period 1 that pay off in state j in period 2. (Note: A state-j Arrow security pays one unit of the consumption good in state j in period 2 but nothing in the other state.) Let q_j be the price of a state-j Arrow security. The consumer's budget constraints in this case are: $c^i_1 = \omega^i - q_1 a^i_1 - q_2 a^i_2$ and $c^i_{2j} = a_j + \epsilon^i_j$, $j = 1, 2$. In equilibrium,

q_1 and q_2 adjust so that the markets for both Arrow securities clear: $a_j^1 + a_j^2 = 0$ for $j = 1, 2$. Compute equilibrium prices and (consumption) allocations for the following parameter configurations:

(i) $\gamma = 1$ (log utility) and $\omega^1 = \omega^2 = 2$

(ii) $\gamma = 3$ and $\omega^1 = \omega^2 = 2$

(iii) $\gamma = 1$, $\omega^1 = 3$, and $\omega^2 = 1$

(iv) $\gamma = 3$, $\omega^1 = 3$, and $\omega^2 = 1$

(b) Now suppose that markets are incomplete: in particular, suppose that consumers are allowed to trade only one asset in period 1, namely, a risk-free bond that pays 1 unit of the consumption good in both states of the world in period 2. In this case, the consumer's budget constraints are: $c_1^i = \omega^i - qb^i$ and $c_{2j}^i = b^i + \epsilon_j^i$, where q is the price of the bond and b^i is the number of bonds purchased by consumer i in period 1. In addition, impose a borrowing constraint that guarantees that consumption in period 2 is nonnegative in both states of the world: $b^i \geq z - 2$. The price q adjusts so that the bond market clears: $b^1 + b^2 = 0$. Compute equilibrium prices and allocations for the four parameter configurations in part (a), assuming that $z = 1/2$. Compare your findings to those in part (a). How does the introduction of incomplete markets affect the equilibrium gross interest rate (i.e., the inverse of the price of a risk-free bond)?

(c) In the economy with incomplete markets, how does a decrease in z from 1/2 to 0 affect the equilibrium interest rate? How does an increase in z from 1/2 to 1 affect the equilibrium interest rate?