Econ 525a (first half)
Fall 2012
Yale University
Prof. Tony Smith

PROBLEM SET #4

Answers to this problem set are due by the beginning of lecture on Wednesday, October 10. Please submit your answers, including computer code, by email to: `tony.smith@yale.edu`.

1. Write a program that uses bilinear interpolation to approximate the function $f(x, y) = \log(x + y^3)$ for $x, y \in [0.1, 1]$. Use 11 equally-spaced grid points in each direction. Create graphs of the approximation error for different values of x and y.

2. Write a program that uses Chebyshev interpolation to approximate the function $f(x) = \log(x)$ on the interval $[0.1, 1]$. How does the approximation error change as the order of the approximation increases from 1 (linear) to 4 (quartic)?

3. Consider a two-period growth model with idiosyncratic shocks to labor productivity. There are two types of consumers, each of whom has measure one-half. Both types of consumers have the same initial wealth ω in period 1. Each consumer's labor productivity in period 2 is random. There are two states of the world in period 2. With probability one-half, type-1 consumers receive shock e^1 and type-2 consumers receive shock e^2; with probability one-half, type-1 consumers receive shock e^2 and type-2 consumers receive shock e^1. Assuming that consumers do not value leisure, total labor supply in period 2 is $L_2 \equiv (e^1 + e^2)/2$.

   Output in period 2 is produced by a profit-maximizing competitive firm with (aggregate) production function $F(K_2, L_2)$, where K_2 is the total amount of capital in period 2. The production function exhibits constant-returns-to-scale and is strictly increasing and strictly concave in both of its arguments.

   Consumers can save in the form of capital and they can purchase state-contingent claims to consumption. They also (inelastically) supply labor in a competitive labor market in period 2. Assuming a complete set of state-contingent claims, a typical consumer's budget constraints read:

$$c_1 = \omega - k_2 - q^1 a^1 - q^2 a^2$$

   and

$$c_2^i = r(K_2, L_2)k_2 + w(K_2, L_2)e^i + a^i,$$

   where c_1 is consumption in period 1, c_2^i is consumption in state i in period 2, k_2 is savings in capital, and a^i is the number of Arrow securities paying one unit of the consumption good in state i (and zero units in state $j \neq i$). The rental rate of capital r and the wage rate w are the appropriate marginal products of the production function; q^i is the price of an Arrow security that pays off in state i. Consumers seek to maximize $u(c_1) + \beta E[u(c_2)]$ and they face a no-borrowing constraint on capital: $k_2 \geq 0$.

The equilibrium conditions are that $K_2 = k_2$ (i.e., total savings in period 2 is the sum of the savings of both types of consumers—which are the same for both consumers since they face symmetric decision problems) and that the markets for the two Arrow securities clear.

(a) Prove that the competitive equilibrium prices and allocations for this economy are the same as those that obtain in an economy without uncertainty (i.e., one in which $e_1 = e_2$).

(b) Now consider an economy with incomplete markets. In particular, assume that consumers cannot trade Arrow securities, but that they can save in the form of capital. Find an equation that determines aggregate savings K_2 in period 2. Is aggregate savings higher or lower than in part (a)? (In the absence of a proof, try a numerical example with logarithmic utility.)

(c) In the economy with incomplete markets, suppose that the government introduces a proportional tax on investment and returns the proceeds to consumers in a lump-sum fashion. The consumer's budget constraints now read:

$$c_1 = \omega - (1+\tau)k_2 + \tau K_2$$

and

$$c_2^i = r(K_2, L_2)k_2 + w(K_2, L_2)e^i.$$

A typical consumer takes the lump-sum subsidy τK_2 as given when making his choices. Prove that increasing τ from 0 reduces aggregate savings. (Hint: Find an equation that determines K_2 implicitly as a function of τ, totally differentiate it with respect to τ, and then evaluate the resulting expression at $\tau = 0$.)

(d) Prove that increasing τ from 0 increases a typical consumer's ex ante welfare. Try to provide intuition for this result. (Hint: You may need to use the fact that if F is homogeneous of degree 1, then $KF_{KK} + LF_{KL} = 0$.)