1. Write a program that uses Chebyshev interpolation to approximate the function \( f(x) = \log(x) \) on the interval \([0, 1]\). How does the approximation error change as the degree of the approximation increases from 1 (linear) to 4 (quartic)? How well does the interpolating polynomial perform outside the interval of approximation? (Recall that Chebyshev interpolation means to construct a polynomial of degree \( n \) that matches \( f \) at \( n + 1 \) grid points that are the roots of the Chebyshev polynomial of degree \( n + 1 \). One way to construct this polynomial is to write it as a linear combination of Chebyshev polynomials, with coefficients calculated using the formulas presented in lecture. Chebyshev polynomials are defined only on \([-1, 1]\), so evaluate them at \( z = 2(x - a)/(b - a) - 1 \), where \( x \in [a, b] \) and \([a, b]\) is the interval of approximation.)

2. Write a program that uses Gauss-Hermite quadrature to compute \( E[e^X] \), where \( X \sim N(\mu, \sigma^2) \). Set \( \mu = 1 \) and \( \sigma = 1, 2, 3 \). How does your answer change as you vary the number of quadrature points from 2 to 10? (You can check your numerical answer either by using the analytical formula \( E[e^X] = e^{\mu + \sigma^2/2} \) or by estimating the expectation using Monte Carlo integration. To obtain the Gauss-Hermite weights and abscissas, you can use the program \texttt{gauher} in Chapter 4.5 of \textit{Numerical Recipes}.)