Markets for Advice

[Preliminary Draft]

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Abstract

The markets for advice are plagued by informational problems. The difficulty of the question posed, the skill of those advising on its answer, and whether a real solution to the problem was actually provided are often hard to assess. We study these problems in a general equilibrium setting, where heterogeneous agents may decide whether to generate productive opportunities or become advisors. When all these circumstances are in place, the market completely breaks down and no advice is sought or given. Entry regulation, where an external exam sets a minimum quality requirement to become an advisor, does prevent the market breakdown. However, entry is (optimally) set too hard relatively to the first best. If, instead, output is transferable (a problem and the good associated with it may be sold for a fee), partial efficiency, this time with too much advice, may be achieved. Finally, if output (whether problems are solved) is verifiable, output contingent contracts are possible. Matching of advisors and advisees is improved, but these contracts may result in ‘too many advisors.’ Even when the first best allocation is achieved, the informational asymmetries generate some income redistribution.

I Introduction

A key role of markets and other institutions is ensuring that society uses optimally the available knowledge (Hayek, 1945). Doing this requires, in particular, that the superior knowledge of experts be conserved for the ‘right’ problems. The solution to this problem takes the form of ‘knowledge-hierarchies’ which allow experts to leverage their knowledge through the use of less expensive workers to deal with routine problems (Garicano, 2000). Firms and other organizations often structure these hierarchies by placing agents in different positions that signal their expertise to others. Often such hierarchies are set up in the market, through consulting and other arrangements that allow experts to advise others. However, these markets are plagued by informational problems. First, the actual difficulty of the problem posed is often hard to assess. Second, the skill of the consultant offering his services in the market is also unobservable. Finally, the output of the advisor (whether the
problem is or not solved) is itself often uncertain or at least ambiguous in nature. As a result, these markets are unlikely to be efficient. Inefficiencies can arise from excessive trade, too little trade, and inefficient matching between problems and experts. We study how different institutions shape the way the market solves this problem and study the efficiency of these different institutions.

We carry out our analysis in a general equilibrium framework since we allow the agents to endogenously choose whether they want to participate in the market for advice or not. Agents, who are heterogeneous in their skill level, must decide if they use their time to originate productive opportunities or if they become advisors and use their time and knowledge to help other agents with their opportunities. The productive opportunities vary in their difficulty: some are harder than others. If an originating agent has a problem that is too hard for him, he might seek the advice of an expert. The market equilibrium determines who should become an advisor and who an originator, what is the reward structure associated with this solution and who should solve problems for whom.

As a benchmark, we first analyze in Section II the market with no informational frictions. In the first best, there is positive sorting - more knowledgeable agents are matched with the harder problems, which are those originated by the most skilled originators.\(^1\) The value of the market to agents is non-monotonic: the market benefits most the most skilled agents, who can solve a lot of problems, and the least skilled agents, who without the market could extract very little value from the problems they originate. The intermediate agents, on the other hand, are not much better off than in autarchy. As originators they ask for advice on problems which are hard to solve and thus require very talented (and expensive) advisors; as problem solvers they cannot solve a large share of the problems on which they were asked for help.

The contractual problems appear because generally agents knowledge and the (expected) difficulty of the problems is unobservable. We first show, in Section B, that if the informational asymmetry is one-sided, the first best can still be attained. It is well known for bilateral relationships that if the party with private information can be made the residual claimant (and if the agents are risk neutral) an efficient outcome can be attained. A similar logic extends to a two sided market. Markets can be set so that prices are based on the observable type - a referral market when the originator’s type is observable or a consulting market when the expert’s type is observable. Equilibrium prices in turn induce the side of the market with private information to self-select the efficient match.

Efficiency is harder to attain when asymmetric information is double sided, that is, neither the knowledge of agents nor the expected difficulty of the problems they pose can be observed. These markets are characterized by double sided adverse selection. In particular, those seeking advice pretend their problems are easier than they are so that they can pay less for advice. On the other hand, advisors want to pretend they are smarter than they are, so that they can get a higher fee. In other words, advisors want to play smart, while originators want to play dumb. Moreover, and further complicating the problem, whether a problem is or not actually solved is often unverifiable.

\(^1\) Garicano and Rossi-Hansberg (2004) introduced and sketched a solution of this full information problem.
Consider, for example, a firm that needs advice on its future strategy—how can the quality of advice given be evaluated?

Unsurprisingly, as we show in Section III, a matching market where neither the quality of sellers nor that of the buyers can be observed works inefficiently. In fact, if the good associated with the problem is not transferable, and output is unverifiable, the market completely breaks down, and no advising actually takes place: the worst agent in the economy can always pass himself for an advisor, and thus advice is worthless. To solve this problem, a certification mechanism able to set minimum standards for access into the role of advisor is necessary. In Section IV we study the functioning of such a market and characterize the optimal mechanisms. In particular, we show that entry regulation involves less advisors than the first best, since those seeking advice are matched with the average advisor in the market, which makes advice is less valuable than under optimal matching.

Depending on the market, some institutional features may alleviate the informational problem described. First, suppose that, even when output is not verifiable, the problems originated are associated with a good, and this good is transferable. In this case, a ‘referral’ market, where advisors pay a fee for unsolved problems, prevents the least skilled agents from entering the market, while allowing the most skilled advisors to buy unsolved problems (or the goods tied to them). This market cannot achieve full efficiency- in particular, the matching of problems to experts is highly imperfect, since contracts cannot be contingent. However, as long as the communication costs is not too high (so that advisors are sufficiently productive), a pooling equilibrium exists in which all problems are sold for the same price.

Second, if it is possible to verify whether the advice given did or did not solve the problem raised, conditional payments (a fee paid only if the problem posed is solved) are possible, and contingent contracts can be used. We study and characterize optimal contracts in this case. Originators pay or receive a fee for the right to a share in case the problem is solved. This, we show, disciplines both originators and problem solvers. The output contingent payment increases with the difficulty of problem to the point that the smartest experts (who tackle the hardest problems) actually become full residual claimants to the output. On the other hand, the price paid by originators is non-monotonic. At first it is increasing since a higher expected payment is needed to attract smarter consultants and the increase in the variable compensation is insufficient to do this on its own. As problems get harder, the variable compensation plays a larger role and hence it is necessary to start compensating the originators for giving up such a large share of their productive opportunity. Hence, not only does the fee decrease but furthermore it becomes negative i.e. the expert starts buying a share of the venture. The non-monotonicity of the payments could lead to two originators paying the same fixed but different shares; separation is ensured because the matching is different-the agent that gives up a higher share is being compensated by being matched with a higher skilled agent.
We show that asymmetric information reduces inequality relatively to a perfect information market. Agents in the middle of the distribution are made better off by the absence of information, as preventing them from imitating others requires that they capture some rents; for the same reason those agents on the extremes are made worse off. This effects are augmented by the entry of more advisors that exists in the second best: the best problem solvers receive harder problems and see their income drop relative to the first best; the worst originators sell their problems to worse problem solvers.

No previous literature has, to our knowledge, examined the double sided adverse selection issue raised by the matching of expert advisors to problems under asymmetric information. The previous literature on expert services has emphasized moral hazard issues involved in the provision of expert services - experts have little incentive to provide the right level of effort. Demski and Sappington (1987), examines the trade-off between productive effort and information gathering incentives by the expert. In Wolinsky (1993) the issue is the incentives of experts to recommend the right treatment - small treatment for small problem and big treatment for large problem. He shows that specialization is optimal in this case. Similarly, Pesendorfer and Wolinsky (2003) study the provision of adequate diagnosis effort by experts. Taylor (1995) studies how insurance can solve informational asymmetries in a context where only the expert can tell the treatment needed. Garicano and Santos (2004) are the only precedent where matching of opportunities and experts take place. They study a problem which combines a (one sided) matching problem with a moral hazard issue: reallocating an opportunity an agent receives opportunity requires compensating the agent who knows about its existence in a way that gives him incentives to retain the opportunity and exert effort on it when he is actually best qualified to deal with it, while giving him incentives to refer the rest. This incentive must be traded off against the risk of moral hazard on the part of the agent receiving the referral - the better the referral incentives the worse the effort incentives. We depart from all of these literature in that we emphasized the double sided of the informational asymmetries - neither the agent knows the quality of the expert, nor the expert knows a priori the difficulty of the problem posed.

A separate literature has studied occupational licensing as a way to regulate the entry into the expert professions. After Arrow (1963) first advanced the hypothesis that entry regulation was a way to protect consumers under asymmetric information on expert quality, Stigler (1971) countered that entry regulations were captured by insiders. A lively academic literature has follows on both sides. Shapiro (1986) argues that licensing provides incentives for human capital accumulation by the expert under moral hazard. Leland (1979) discussed entry requirements in a market with asymmetric information about quality, and showed that if insiders were in charge, the standard would be set too high. We show that in fact under asymmetric information licensing requirements must be more restrictive than the first best, even absent pressures from insiders. Moreover, in Leland 2Friedman and Kuznets (1946) first argued that licencing far from helping consumers resulted in higher prices and lower quantity and quality of service. Empirical support for this view of licencing as an inefficient regulation has been found in accounting (Young, 1988), dentistry (Kleiner adn Kudrle, 2000) and optometry (Haas Wilson, 1986).
the set of agents who may be experts is given, and there only can be exit from it. Our agents may be in one or the other side, and thus they may enter or exit from expert markets. Because of this difference, while in Leland (1979), like in the original Akerlof (1974) paper, the market may or may not disappear under asymmetric information, our market necessarily breaks down under two sided asymmetric information whenever output is not verifiable.

Finally, our paper fits within a literature studying trade in markets with bilateral asymmetric information. Most of the literature develops from the original Myerson and Satherwaite’s (1983) analysis of trade mechanisms under asymmetric information about buyer and seller valuation, but with multiple buyers and sellers of a commodity for which they have unknown valuations (e.g. Lu and Robert, 2001); they do not care about each other’s types generally, but only about the value of the object at stake and thus matching is irrelevant. The only paper we are aware that studies equilibrium in matching markets with two-sided incomplete information is Gale (2001). He studies it in a more general context and provides restrictive conditions under which separation will take place. Because our problem has inherently much more structure we are able to go substantially further in characterizing the market equilibrium.

II The Model and First best

There is a continuum of income maximizing agents who are indexed by their level of knowledge $z_i \in [0,1]$. Without loss of generality, we choose the index $z$ so that $z_i$ is measured in percentiles of the knowledge distribution – the distribution of $z$ is thus uniform. Agents must first decide if they become problem originators or problem solvers (advisors). We will let $O$ denote the set of agents that become originators and by $S$ the set of agents that become solvers. If an agent is an originator then at the beginning of the period he draws a problem, with an associated difficulty level $q$; $q$ is unobserved and i.i.d. across problems and distributed according to $F(q)$, a continuous function with density $f(q)$. If $z_i > q$ then an agent can solve the problem by himself and he gets a payoff of 1. If $z_i < q$ then he cannot solve the problem. He can then seek an advisor who can potentially solve the problem for a fee. Since not all originators need to seek advice with their unsolved problems we will denote by $A$ the set of agents that do seek advice and by $I$ the set of agents that remain independent.

The other option is for the agent to decide to become an advisor. Advisors can advise up to $1/h$ agents on their problems, where $h < 1$ is the helping cost (it costs a fraction $h$ of time to help one other agent) in a given period. Advisors don’t generate any problems of their own, that is, they are specialized in solving problems. Like originators, advisors can solve problems that are not too hard for them i.e. $z_i > q$. We will assume for simplicity that if neither the original agent nor the hired consultant can solve the problem then the problem goes unsolved.\(^3\)

\(^3\)See Garicano (2000) and Garicano and Rossi-Hansberg (2006) for a setting (without asymmetric information) in which several layers of advice are available, with homogeneous and with heterogenous agents respectively. In those
To represent the (potentially random) matching between advice seekers and solvers we will use the CDF \( M(s, z) \). That is, \( M(s, z) \) will determine with what probability agent \( z \in A \) gets matched with \( s \in S \). We will use \( \mu(s, z) \) to denote its density. In some cases the the matching will not have any random component and \( M(s, z) \) will hence be degenerate. In these cases will simply use the matching function \( m(z) : A \rightarrow S \).

**The Information Structure**  Our objective is to characterize the optimal contracts and division of labor that will arise in this economy. This will depend critically on the assumptions we make about what is observable and what is verifiable. We study the following environments:

- **Perfect Information**: The knowledge of all agents is observable.
- **Unobservable knowledge (Own knowledge is private information)**
  - Non-Verifiable output
  - Output Verifiable

### A Full Information Benchmark

**Properties of the First Best Allocation**  We start by studying the first best. Suppose that a social planner could allocate optimally agents into those who seek advice, those who neither seek nor give advice, and advisors, and could choose which type of advisors help with which problems.

The planners objective can be written as:

\[
\max_{O,A,M(s,z)} \int_{z \in A} (F(z) + (1 - F(z)) \Pr(q < s | q > z, M(s, z))) dz + \int_{z \in I} F(z) dz
\]

With full information, the only constraint that the planner faces is the resource constraint that the demand for advice be not larger than the supply. Formally, for any subset \( D \in A \):

\[
\int_{z \in D} (1 - F(z)) dz \leq \int_{z \in D} \frac{\int_{s \in S} \mu(s, z)}{h} dz
\]  

(1)

Intuitively, it seems clear that, more skilled originators must ask questions to more skilled advisors, and that advisors should be those more skilled at problem solving. We show below that this is indeed the case. (The proofs are in the appendix)

**Lemma 1 (Assortative Matching)**  Let \( s \) be an expert who is solving problems posed by originator \( z \) and \( s' \) one who is solving problems posed by originator \( z' \). If \( z > z' \) optimality requires that \( s > s' \).

papers \( z \) is a choice of agents, whereas here it is given.
To illustrate why the planner would choose assortative matching consider the highest originator type. If he couldn’t solve the problem then the problem is fairly hard in expectation. In contrast, the unsolved problem from the lowest originator is fairly easy in expectation. Assigning the smartest consultant to the easy problem is inefficient. Most likely a less able expert could handle this problem and his time would be better spend solving those problems that are hard in expectation and hence less able experts have a lower probability of solving.

To show that some matching always takes place in equilibrium, consider a situation where all workers are unmatched, the least skilled agent \( z = 0 \) produces \( F[0] = 0 \), while the most skilled agent produces \( F[1] = 1 \). Now consider the value of the match between the best and worst workers. This value is \( \frac{1 - F(0)}{h(1 - F(0))} = \frac{1}{h} > 1 \) as long as \( h < 1 \), and thus this match is welfare increasing.

Assortative matching together with the fact that it is never optimal to have an under utilized expert implies that with full information \( M(s, z) \) is degenerate and hence we can focus on characterizing the matching function \( m(z) \). Before doing so it is useful to establish the following two lemmas:

**Lemma 2 (Independents are Smart)** Suppose there are originators with ability \( z \) that do not seek advice. Then, there cannot be any originator \( z' > z \) that seeks advice when he cannot solve a problem. If \( z \in I \) then if \( z' > z \) \( z' \notin A \).

Intuitively, if some problems are going to be passed on, they must be the easier ones—those have the highest likelihood of being solved by a problem solver. If a problem is too hard to be passed on and is dropped, then all the problems originated by smarter agents are even harder and hence are not worth being passed on to the experts.

**Lemma 3 (Experts are Smarter)** Agents who become experts are smarter than those who become originators. If \( z \in S \) then for all \( z' \in O, \ z > z' \).

It is easy to see that if an agent is an advisor, then someone smarter than him should not be an originator. If this were not the case, then the roles can be swapped and output increased, so that the originator solves the problems that were previously solved by the advisor, but with a higher probability. The gain is larger than the loss, since each advisor solves multiple problems.

The ordering implied by the two lemmas above together with the fact that \( M(s, z) \) is degenerate allows us to write the resource constraint on advising time (eq. 1) as:

\[
\text{for all } z : \int_{0}^{z} (1 - F(q)) \, dq = \int_{m(0)}^{m(z)} \frac{1}{h} \, dt
\]

Equivalently, the integral equation above can be written as:

\[
m(z) = m(0) + h \int_{0}^{z} (1 - F(q)) \, dq
\]
and therefore:

\[ m'(z) = h(1 - F(z)) \]

We summarize our results in the following Proposition:

**Proposition 1** The first best allocation can be characterized by a matching function \( m(z) \) and 2 cutoff types \( z_1 \) and \( z_2 \) where \( z_1 \leq z_2 \). Types \( z \in [0, z_1] = A \) are originators who seek advice, types \( z \in (z_1, z_2) = I \) originate problems but do not seek advice and \( z \in [z_2, 1] = S \) are advisors. The matching function satisfies: \( m(0) = z_2, m(z_1) = 1 \) and \( m'(z) = h(1 - F(z)) \).

This follows from the lemmas above and the need to guarantee the right proportion of consultants to originators so that demand for consultants and their available time are equal type by type.

![Diagram](image)

**Solving for the first best allocation.** Given the characterization provided in the Proposition above, the only thing that remains to be determined are the optimal cutoff types \( z_1 \) and \( z_2 \). In fact, the boundary conditions \( m(z_1) = 1 \) and \( m(0) = z_2 \) imply that:

\[
\int_0^{z_1} (1 - F(z)) \, dz = \frac{1 - z_2}{h}
\]

Hence, the planner can only choose \( z_1 \). This in turn determines the cutoff type \( z_2 = Z(z_1) = 1 - h \int_0^{z_1} (1 - F(z)) \, dz \) and the matching function \( m(z; z_1) = 1 - h \int_0^{z_1} (1 - F(z)) \, dz + h \int_0^{z} (1 - F(q)) \, dq \).

Given the previous results solving for the first best allocation reduces to solving the following:

\[
\max_{z_1} \int_0^{z_1} F(m(z; z_1)) \, dz + \int_{z_1}^{Z(z_1)} F(z) \, dz
\]

Taking the derivative with respect to \( z_1 \) and grouping the terms to facilitate the interpretation, the FOC can be written as:

\[
- \left( \int_0^{z_1} f(m(z; z_1)) \frac{\partial m(z; z_1)}{\partial z_1} \, dz + F(Z(z_1)) \frac{\partial Z(z_1)}{\partial z_1} \right)
= \frac{F(m(z_1; z_1)) - F(z_1)}{Extra \ output}
\]

The condition can be readily interpreted. As \( z_1 \) increases, the marginal gain (the second line of expression 3) more output is produced as more agents are able to seek advice, and produce with
probability $F(m(z_1))$ instead of working on their own and producing with probability $F(z_1)$. There are two sources of losses. First, as more agents are asking for advice, the quality of advice each agent receives is reduced. The reason for this is that with positive sorting, as more agents become advisors, the worst advisor (the one who advises the worst originator), is lower quality, and so on for all originators. The final loss is the output loss from those originators who were self employed and now, instead of generating productive opportunities on their own they provide advise to other agents.

Proposition 2  *The number of independent agents is increasing in $h$. If $h > h'$ then $I' \subseteq I$.  

Suppose problems are distributed uniformly then can replace in the first order conditions:

\[
m(z; z_1) = 1 - h \left( z_1 - z - \frac{z_1^2 - z^2}{2} \right) \\
Z(z_1) = 1 - h \left( z_1 - \frac{z_1^2}{2} \right)
\]

And the following figure presents the solution for this uniform case for each value of the parameter $h$.

As can be seen from the figure above, the number of experts is non-monotonic in the number of problems an expert can address. When the experts cannot leverage their expertise with many

Figure 1: First best allocation of agents as a function of communication costs $h$. 
problems \((h \text{ close to } 1)\) then it is not worth having knowledge hierarchies and most agents are independents. As \(h\) decreases it is more efficient to have experts. The number of independents decreases monotonically. For the uniform case, for \(h \leq 0.75\) it is efficient not to have any independents at all. From this point, if we continue to increase the number of problems an expert can address (decreasing \(h\)) the number of experts starts falling. This is simply because fewer experts are needed to address all the unsolved problems.

**B Competitive Equilibrium and One Sided Informational Asymmetries**

With perfect information, the first best can be attained in a decentralized way as a competitive equilibrium. In fact, there are many different decentralizations that can implement the first best; they are all equivalent in the allocation they support, which is unique. In particular, we will consider specifically two decentralizations, which are readily interpretable and will be useful later on. They differ in the agent who obtains the residual income from solving the problem. As a result, they deal differently with asymmetric information.

Letting the advisor of an agent \(z\) be \(m(z)\), the joint output that a matched pair produces is given by:

\[
F(z) + (1 - F(z)) \frac{F(m(z)) - F(z)}{(1 - F(z))} = F(m(z))
\]

That is, with probability \(F(z)\) the originator produces on his own, and with probability \((1 - F(z))\) he needs help. Thus the (ex post) output of the match (conditionally on advice being needed) is given by \(\frac{F(m(z)) - F(z)}{(1 - F(z))}\) per worker or, given that a problem solver may have \(1/h\) originators, \(y = \frac{F(m(z)) - F(z)}{h(1 - F(z))}\). This function displays increasing differences

\[
\frac{\partial^2 y(z; z_s)}{\partial z \partial z_s} > 0,
\]

so the competitive equilibrium must be characterized by positive sorting, \(m'(z) > 0\). The competitive equilibrium must result in occupational choices for all agents among originating or advising, in an earnings stream for originators and solvers, and in an allocation of originators to advisors (a matching function). The two decentralizations differ in who claims the residual income from the problem potentially being solved. We define them next.

**Definition 1** In a consulting market originators pay a fixed price for advice \(w(z)\) and claim the residual income from the problem solution. Earnings of originators \(z\) who hire advisors \(z_s\) are \(W^c_o(z; z_s) = F(z) + (1 - F(z)) \left( \frac{F(z_s) - F(z)}{1 - F(z)} - w(z_s) \right)\), while earnings of advisors of skill \(z\) are \(W^c_s(z) = \frac{w(z)}{h}\).

**Definition 2** A referral market has advisors claiming the residual income from the problem solution; they pay a fixed price \(r(z)\) in exchange of the problem. Earnings of originators \(z\) are then \(W^r_o(z) = F(z) + (1 - F(z))r(z)\); those of advisors who buy problems from originators of skill \(z_o\) are \(W^r_s(z; z_o) = \frac{1}{h} \left( \frac{F(z) - F(z_o)}{1 - F(z_o)} - r(z_o) \right)\).
We proceed now to characterize the equilibrium in each of these markets. We will show that the allocations and earnings are identical, and identical to the first best.

**A Consulting Services Market**

In a consulting services market originators hire advisors of skill \( z_s \) for a fixed fee \( w(z_s) \). Originators remain the residual claimants to output. Earnings of advisors do not depend on who they match with; their earnings are simply determined by the equilibrium consulting fee (they make no choices):

\[
W(z) = \frac{w(z)}{h}
\]

on the other hand, originators earn the residual, so they care directly about the choice of partner:

\[
W_{co}(z; z_s) = \max_{z_s} F(z) + (1 - F(z)) \left( \frac{F(z_s) - F(z)}{1 - F(z)} - w(z_s) \right)
\]

With first order condition for the optimal choice of advisor:

\[
f(z_s) - (1 - F(z))w'(z_s) = 0
\] (4)

Before characterizing the competitive equilibrium, note that since \( w'(z_s) \) must be increasing in equilibrium, \( \partial^2 W_{co}(z; z_s)/\partial z \partial z_s > 0 \), and the matching function \( z_s = m(z) \) must be increasing, \( m'(z) > 0 \). The competitive equilibrium in this case can be characterized as follows:

**Definition 3** A competitive equilibrium in a consulting service market is a fee schedule \( w(z) \) paid for by problem originators to advisors, a matching function \( m(z) : O- \rightarrow A \) allocating advisors to originators; (2) a pair of cutoffs \( \{z_1, z_2\} \), such that \( O = [0, z_1] \) is the set of originators who seek advice, \( I = [z_1, z_2] \) is the set of originators who do not seek advice (independent) and \( A = [z_2, 1] \) is the set of problem solvers; (3) and an earnings function \( W(z) \), such that: (1) Supply equals demand point by point; (2) the matching is such that no originator can do better by choosing a different consultant; (3) No agent can be made better off by an occupational (from originator to advisor) change or by deciding to seek or forgo advice.

To construct the equilibrium, start from the supply and demand conditions. Supply equals demand pointwise implies: \( m'(z) = (1 - F(z))h \). With \( m(0) = z_2 \), we can write the matching function as \( m(z; z_2) \). Then for a given \( z_2 \), the matching function evaluated at the the highest originator is:

\[
m(z_1; z_2) = 1
\]

implies that the match is entirely pinned down up to one constant \( z_1 \). Trivially, \( m(z_1; z_2) = 1 \) implies a function \( z_2^{sd}(z_1) \) with \( z_2^{sd} < 0 \) (intuitively, if the supply of problems requiring advise increases \( -z_1 \) goes up– you need more problem solvers – \( z_2 \) must decrease).
Notice also that the first order condition (4) must hold for all \( z \). Thus given some matching \( m(z; z_2) \), the first order condition determines a wage function for each \( z_2 \).

\[
w'(z; z_2) = \frac{f(z)}{(1 - F(m^{-1}(m(z; z_2))))} \tag{5}\]

This differential equation can be solved simply by integration, as there is no \( w(,) \) on the right hand side, and generates a wage function \( w(z; z_2) \). To solve for the constant of integration, use \( \frac{1}{h} w(z_2; z_2) = F(z_2) \). Finally, optimal occupational choices also requires that the top originator be indifferent between seeking or not advice: \( W_{c1}(z_1; 1) = 1 - (1 - F(z_1))w(1; z_2) = F(z_1) \), which implies \( w(1; z_2) = 1 \). This allows us to solve for \( z_2 \). The following proposition summarizes this analysis (see the Appendix for a detailed proof).

**Proposition 3** A consulting services market, in which originators pay for advice from consultants as needed, achieves the first best. The competitive equilibrium is unique.

We show that the competitive equilibrium achieves the first best in the appendix. Note that nothing in the argument above requires that we observe the ability of the originators. The consultants do not make any choice, so they do not need to observe anything. Thus suppose that in fact, the originator skill is unobservable, but the skill of consultants is not. This could be the case, for example, if consultants have developed a reputation that allows agents to know who is knowledgeable and who is not, while problem originators are unknown, and so are their types. In this case, the consulting market we have just described would work exactly in the same way we suggested. We state this in the following corollary.

**Corollary 1** Under one sided asymmetric information, where only the consultant skill can be observed but not the originator skill, the consulting service market still attains the first best.

**A Referral Market**

In a referral market originators transfer the whole residual ownership of the problem to advisors, in exchange for a fixed referral price \( r(z) \). The earnings of originators, for a given per problem price, are given– originators now do not need to choose anything:

\[
W_{o}^r(z) = F(z) + (1 - F(z))r(z) \tag{6}
\]

While advisors earnings are a function of whom they choose to buy problems from:

\[
W_{s}^r(z; z_o) = \max_{z_o} \frac{1}{h} \left( \frac{F(z) - F(z_o)}{1 - F(z_o)} - r(z_o) \right) \tag{7}
\]

The optimal choice of \( z_o \) by an advisor problem solver with skill \( z \) requires:

\[
\frac{f(z_o)(1 - F(z))}{(1 - F(z_o))^2} = r'(z_o) \tag{8}
\]
Again, note that as in the first best, and in the consulting services market, the competitive equilibrium must be characterized by assortative marching since $\frac{\partial^2 w(z_1)}{\partial z_1 \partial z_2} > 0$. We can define the competitive equilibrium analogously to the consulting case.

**Definition 4** A competitive equilibrium in problem referrals is a price schedule $r(z)$ paid by advisors in exchange for a problem, a matching function $m(z): O\rightarrow A$ allocating advisors to originators; (2) a pair of cutoffs $\{z_1, z_2\}$, such that $O = [0, z_1]$ is the set of originators who seek advice, $I = [z_1, z_2]$ is the set of originators who do not seek advice (independent) and $A = [z_2, 1]$ is the set of problem solvers; (3) and an earnings function $W(z)$, such that: (1) Supply equals demand point by point; (2) no consultant can do better by choosing to buy problems from a different originator; (3) No agent can be made better off by an occupational (from originator to advisor) change or by deciding to seek or forgo advice.

The first part of the equilibrium construction, using the supply equal demand condition, leads to the same function $m(z; z_2)$ and the supply and demand condition result in a downward sloping marginal manager function $z_{sd}^2(z_1)$. Substituting again in the first order condition, we have

$$-\frac{f(z)(1 - F(m(z; z_2))}{(1 - F(z))^2} = r'(z)$$

Which we can integrate for each $z_2$ to obtain a function $r(z; z_2)$ and a constant. Again we can solve for the constant by using $W^*_r(z_2) = \frac{1}{k} (F(z_2) - r(z; z_2)) = F(z_2)$ so that $r(z, z_2) = F(z_2)(1 - h)$. And we can finally, as previously, obtain a second condition for $z_1$ and $z_2$ by using the indifferece condition of the top originator between seeking or not advice: $W^*_c(z_1; 1) = F(z_1) + (1 - F(z_1)) r(z_1) = F(z_1)$ thus $r(z_1; z_2) = 0$. This generates a condition $z_{sd}^2(z_1)$ with $z_{sd}^2(z_1) > 0$, as we show in the appendix. Moreover, occupational choice is optimal. The following proposition summarizes this analysis (see the Appendix for a detailed proof).

**Proposition 4** A referral market, in which consultants pay for unsolved problems from originators, achieves the first best. The competitive equilibrium is unique. Moreover, the equilibrium allocation and earnings in the referral market are the same as in the consulting market. The competitive equilibrium allocation is unique.

Similarly to the consulting market, nothing about the equilibrium in this market requires observing the skill of consultants. This means that a referral market can achieve the first best in a situation in which the consultant skill is unobservable. For example, suppose all agents can see the skill of agents less skilled than themselves. Then one sided asymmetric information follows. In this case, having the informed side, the advisors, be the buying side, results in the first best.

**Corollary 2** Under one sided asymmetric information, where only the skill of originators can be observed (for example, all agents can observe the skill of those less skilled than themselves) the referrals market still attains the first best.
Thus straightforward institutional arrangements can achieve efficiency if the informational problems are only one sided. In general, in bilateral relationships making the party with private information the residual claimant allows for efficiency. We have shown that a similar logic extends to this two sided market. As long as the market is set up so that prices are based on the observable type—a referral market when the originator’s type is observable or a fee based market for advice when the expert’s type is observable, equilibrium prices will induce the side of the market with private information to self-select the efficient match.

**Who gains most from the market for advice?**

Inspection of figure (2) clarifies an important intuition on the value of skill in this market. The agents who gain most from being able to give and ask for advice are those in the extremes: the best problems solvers and the worst originators. In a way, the better the originator the worse the quality of the goods he sells, in the sense that he is seeking advice on harder problems. Thus ‘quality’ is decreasing in skill on the advice seeking, side of the market. In the margin, originator $z_1$ is indifferent between seeking advice or not- to him, advice has no value– his earnings are the same with or without it. On the other side of the market, quality increases in skill- the better the agent, the better advice he can give; thus he benefits most from the market for skill.

**III Two-Sided Asymmetric Information**

We now turn to the case in which the agents types are their private information. This becomes a trading problem with two-sided adverse selection. Consultants might want to pretend they are smarter than they truly are and originators might want to pretend that their unresolved problems are simpler than they really are.
We first analyze the case in which output is unverifiable and ownership is non transferable. The only type of market which could be set up in this case is one with uncontingent wages in exchange for expert services. This market breaks down because at wages high enough to motivate high types to become experts low types would want to enter the expert sector.\footnote{A potential solution to this market breakdown is the use certification which we study in Section IV.}

Next we look at the case in which although output is unverifiable productive opportunities can be transferred to the experts making them full residual claimants. In this case there will be competitive equilibria with trade. In fact, there will be too much trade relative to first best. Problems that a planner would not have transferred to an expert in the first best will be traded in the market in this case.

Finally, we consider the case in which output is verifiable and hence fully contingent contracts can be written. In this case, for low values of $h$ the planner can achieve the first best allocation.

\section*{A Unverifiable output and non-transferable ownership}

When output contingent contracts cannot be written and ownership cannot be transferred, the originators are full residual claimants and experts can only be paid an uncontingent fee. Since their payoff is uncontingent all experts must receive the same payment. Under these circumstances, the market breaks down completely. No trade can take place, as the lowest skilled agents in the economy can pretend to be smarter and become sellers of consulting services. Any fixed fee that is high enough to entice a highly skilled agent to become a consultant will induce the least skilled agents to misrepresent their knowledge and offer their "services" for this fee.

\textbf{Proposition 5} \textit{When output contingent contracts cannot be written and ownership cannot be transferred there cannot be a competitive equilibrium with trade of expert services.}

The intuition for this result is that the expected earnings of becoming an originator depend on the agent’s type but the expected earnings of becoming an expert (or pretending to be one) are independent of type. Hence, if there is some type that prefers to become a consultant then all types below want to follow the same path.

Note that in contrast to the classic lemons problem like in Akerlof (1970) the main reason for the market to break-down is not coming from an unravelling from high types exiting but rather from the excessive entry of low types. To illustrate, consider the market for brain surgeons. The fact that top brain surgeons are not differentially compensated from good brain surgeons is second order. The first order problem arises from the average Joe putting a white robe and offering to crack your head open. Hence, to be able to get the market operating, we must find a way to prevent the low types from becoming false experts. We explore this idea in Section IV.
B Unverifiable output with transferable ownership

In some instances, the productive opportunities may be transferred. Consider, for example, a lawyer who has a client with a hard problem. Even when the successful resolution of a problem is itself unverifiable, the client can be transferred to another lawyer, in exchange for a fee. If ownership is transferable in principle there could be two types of contracts. One in which the originators are left as residual claimants and one in which the opportunity is sold to the experts and hence they are made residual claimants. We will use $w$ to denote the fee paid for consulting services and $r$ for the referral fee paid to take ownership of a productive opportunity.\footnote{In principle we could allow for greater generality by allow the offers to be type specific i.e. $w(z)$ and $r(z)$ but given that types are private information it is easy to show that it is without loss to simply consider one wage and one referral price.} We will denote the set of agents that become originators and hire consultants by $A_w$ and those that sell their productive opportunities by $A_r$. As before, the set of those originators who do not seek advice nor sell their problems will be denoted by $I$. Similarly for the solvers we will denote by $S_w$ those who sell their services and by $S_r$ the set of those who purchase productive opportunities.

**Definition 5** A Competitive Equilibrium with transferable ownership consists of prices $w$ and $r$, sets $A_w$, $A_r$, $I$, $S_w$ and $S_r$ and a matching distribution $M(s, z)$ that determines who trades with whom. Such that: i) Agents maximize in their choice of occupation and in their choice of contract to use. ii) Markets clear.

We first show that in all competitive equilibria $S_w = \emptyset$. That is, there cannot be any experts selling their services for a fixed fee in equilibrium.

**Lemma 4** There cannot be any experts selling their services in any competitive equilibrium i.e. $S_w = \emptyset$.

The proof follows similar arguments to those in Proposition 5, low types would have an incentive to become experts if they can do so without becoming residual claimants. Therefore, all experts will have to become full residual claimants.

If $r > 0$ there cannot be any independents, $I = \emptyset$. Any $z \in I$ would rather sell his productive opportunity in the market and collect $r > 0$ so there cannot be any types left out of the market. In this case, market clearing conditions imply a unique cutoff type $z^*$ such that $A_r = [0, z^*]$ and $S_r = [z^*, 1]$. $z^*$ must therefore satisfy:

$$
\frac{1 - z^*}{h} = \int_0^{z^*} (1 - F(q)) dq
$$

Note that the demand is decreasing in $h$ so $z^*$ must be decreasing in $h$ as well.\footnote{Note that demand is decreasing in $z^*$, and supply is increasing in $z^*$. Moreover, if $z^* = 0$, there is excess demand, while if $z^* = 1$ there is excess supply.}
Given $z^*$ the referral price $r$ must satisfy that type $z^*$ be indifferent between becoming an originator or an expert:

His expected earnings if he becomes an originator are:

$$w_o(z^*, r) = F(z^*) + (1 - F(z^*)) r$$

the first term are the earnings if he solves the problem himself the second term are the earnings if he passes it to an expert and gets $r > 0$ in exchange.

The expected earnings of type $z^*$ from becoming an expert are given by:

$$w_e(z^*, r) = (E_x \Pr [q < z^*|q > x] - r)$$

Since,

$$E_x \Pr [q < z^*|q > x] = \frac{\int_0^{z^*} \left( \int_x^{z^*} f(q) dq \right) dx}{\int_0^{z^*} \left( \int_x^{1} f(q) dq \right) dx} = \frac{\int_0^{z^*} (F(z^*) - F(x)) dx}{\int_0^{z^*} (1 - F(x)) dx}$$

$$w_e(z^*, r) = \frac{\int_0^{z^*} (F(z^*) - F(x)) dx}{\int_0^{z^*} (1 - F(x)) dx} - r$$

Since $w_o(z^*, r)$ is strictly increasing in $r$ and $w_e(z^*, r)$ is strictly decreasing in $r$ we can find the unique $r^*$ that guarantees that $w_e(z^*, r^*) = w_o(z^*, r^*)$.

$$r^* = \frac{\int_0^{z^*} (F(z^*) - F(x)) dx}{\int_0^{z^*} (1 - F(x)) dx} - hF(z^*)$$

For the uniform case we can obtain close form solutions. The market clearing condition (eq. 10 above) determines:

$$z^* = 1 - \frac{1}{h} (\sqrt{h^2 + 1} - 1)$$

The indifference condition for type $z^*$ yields a referral price of:

$$r^* = 2 - \frac{1 + 2h}{\sqrt{h^2 + 1}}$$

In the expression above, $r$ is strictly decreasing in $h$ and for $h = \frac{3}{4}$, $r = 0$. Therefore, for the uniform case, an equilibrium with $r > 0$ only exists for $h \leq \frac{3}{4}$.

With $r = 0$ there could still be equilibria in which some agents decide to become experts. In fact, there is a continuum of such equilibria. In all of them, there is an excess supply of problems. These equilibria differ in which problems get transferred at a price of zero. The worse the transferred
Figure 3: Equilibrium in Uniform case. For $h > .75$ the price of problems is 0. In this case there always is excess supply of problems, and the decision to enter into consulting depends on the selection of problems that do get transferred. Maximum entry (lower bound) occurs when the easiest problems get transferred; minimum entry (upper bound) when only the hardest problems get transferred.

problems are, the lower the entry into the expert market. We can find an upper and lower bound to the entry into the advisory market by assuming that the problems from the dumbest (smartest) originators are passed on to the experts. Letting the interval of consultants be $[z_2, 1]$, the easiest case corresponds to the case where the problems are in the interval $[0, \bar{z}]$ with $\bar{z} < z_2$; the hardest case when the problems are in $[\bar{z}, z_2]$.

For example, for the uniform case and in the intermediate case in which problems are drawn randomly from the pool of unsolved problems:

$$w_e(z^*, 0) = w_o(z^*, 0)$$

$$z^* = 2 - \frac{1}{h}; \ h > \frac{3}{4}$$

Figure 3 illustrates the properties of the equilibria for the uniform case, with the bound on the worst advisor ($z^*(h)$) for each $h$ depending on whether the problems selected are the worst problems, a random selection of problems, or the best problems drawn by originators.

The following proposition summarizes the discussion above.

**Proposition 6 (Excessive Advice)** Under no information, when agents can be made residual claimants for the problems, there exists an $h^* \in (0, 1)$ such that

i) For any $h \leq h^*$ a unique competitive equilibrium with referrals exists, in which agents, $z \in [0, z^*(h)]$ are problem originators and agents $z \in [z^*(h), 1]$ are problem solvers, and there is constant price $r^*(h)$ per problem referred. The cutoff $z^*(h)$ is strictly decreasing in $h$ and is larger
than the first level cutoff $z_1$.

ii) For any $h > h^*$ there exist a continuum of equilibria with $r = 0$. The equilibria differ in the value of $h$ and the distribution of problems that are passed on to experts. Both the lower bound and upper bound of equilibrium values for $z$ are increasing in $h$ and as $h \to 1$ there is a unique equilibrium with $z^* = 1$.

Thus, while the referral market does not break down, it suffers from three types of efficiency losses with respect to the full information problem; first, problems that are too hard to be referred are in the pool of problems passed on; second, there is too much entry into consulting; and third, there is inefficient matching- conditional on a problem being passed, the probability that it is solved is much lower, as the matching is now random instead of assortative.

C Verifiable Output: Contingent Contracts

Suppose now that it is possible to contract on output. Agents can pay conditional on the solution being found. Without loss of generality, contracts can be characterized by two parameters $w, \alpha$. $w$ being the uncontingent payment to the problem solver and $\alpha$ the additional payment to the problem solver is he succeeds in solving the problem. Note that $w < 0$ would correspond to the advisor paying for giving advice- purchasing, in a way, the problem, in exchange for a share of the output.

We construct a separating equilibrium in which each type of originator offers a different contract and each type of solver works for a different originator, that is, the matching function is strictly monotonic. Let $\omega_x = \{w_z, \alpha_z\}$ denote the contract offered by type $z$ and $m(z)$ denote the solver type that attempts to solve a problem originated by type $z$. We first show that the equilibrium must exhibit positive assortative matching.

Lemma 5 Any separating equilibrium must exhibit positive assortative matching, $m'(z) > 0$.

Since $m(z)$ is strictly increasing market clearing type by type essentially pins down $m'(z)$. The only degree of freedom left comes from how many agents become originators. We will therefore denote the matching function by $m(z; z_1)$ to explicitly capture this. In principle, there will be two possibilities either, types $z \in [0, z_1]$ will be originators, $[z_1, z_2]$ independents and types $z \in [z_2, 1]$ will be solvers; or $z_1 = z_2 = z^*$, where then $z \in [0, z^*]$ are originators and $z \in [z^*, 1]$ are solvers- no agents are independent. In next Lemma we show that there does not exist a separating equilibrium where some agents do not seek advice, and hence we focus on the case where all agents seek advice.

Lemma 6 In any separating equilibrium all originators must seek advice in equilibrium.

Intuitively, a separating equilibrium with some problems unsolved must keep some agents out of the market. This restricts the prices to be positive $w > 0$ (since otherwise anyone not seeking advice

\footnote{We delay the formal definition of the equilibrium since it is convenient to establish first that all originators must seek advice in equilibrium. A general definition can be found in the Appendix.}
could always receive a payment by entering the market place) which limits excessively the space of available contracts.

Given the two Lemmas above, we can conclude that there can be at most one separating equilibrium.

**Proposition 7** There is at most one separating equilibrium.

The qualifier "at most" is used in the proposition above because for high values of $h$ there might be no separating equilibrium. Below we construct the equilibrium for the case in which problems are uniformly distributed, $F(q) = q$.

**Originators and Problem Solvers Problem.** Let $V(z, \tilde{z})$ denote the value (ex-post) of an originator of type $z$ who failed to solve his problem from pretending to be type $\tilde{z}$:

$$V(z, \tilde{z}) \equiv -w_{\tilde{z}} + (1 - \alpha_{\tilde{z}}) \Pr(q < m(\tilde{z}; z_1) | q > z)$$

$$\equiv -w_{\tilde{z}} + (1 - \alpha_{\tilde{z}}) \frac{m(\tilde{z}; z_1) - z}{1 - z}.$$

Hence, we can define the ex-ante expected value of becoming an originator and pretending to be type $\tilde{z}$ if trading with an expert:

$$R(z, \tilde{z}) \equiv z + (1 - z) \max \{V(z, \tilde{z}), 0\}$$

Let $S(z, \tilde{z})$ denote the value of a problem solver of type $z$ who pretends to be $\tilde{z}$ and thus buys problems from type $m^{-1}(\tilde{z})$:

$$S(z, \tilde{z}) \equiv \frac{1}{h} \left( w_{m^{-1}(\tilde{z}; z_1)} + \alpha_{m^{-1}(\tilde{z})} \Pr(q < z | q > m^{-1}(\tilde{z}; z_1)) \right)$$

$$\equiv \frac{1}{h} \left( w_{m^{-1}(\tilde{z}; z_1)} + \alpha_{m^{-1}(\tilde{z})} \frac{z - m^{-1}(\tilde{z}; z_1)}{1 - m^{-1}(\tilde{z}; z_1)} \right).$$

**Equilibrium Contracts and Matching**

**Definition 6** A set of contracts $\omega_z$, a matching function $m(z)$ and a cut-off for occupational choice $z^*$ are a separating equilibrium if: (i) demand for advice equals supply of expert services; (ii) only those agents $z > z^*$ choose to become expert advisors; (iii) agents with $z < z^*$ choose to seek help with their unsolved problems; (iv) both types of agents truthfully reveal their types.

(i) **Supply and Demand.** Given that there is assortative matching, the matching function is like in the first best, that is it is given by (2) up to the parameter $z^*$. In particular, given the uniform problem density, $z^* = \left(1 - \frac{1}{h} \left(\frac{h^2}{h^2 + 1} - 1\right)\right)$ and the matching function is then:
\[ m = h z - \frac{1}{2} h z^2 + \left( 1 - \frac{1}{h} \left( \sqrt{h^2 + 1} - 1 \right) \right) \]  

(11)

(ii) **Occupational Choice.** We need that each type chooses the right occupation.\(^8\)

For advisors \((z > z^*)\) not to prefer to originate their own problems,

\[ S(z, z) \geq R(z, \tilde{z}) \quad \text{for all } \tilde{z}; \]

and for originators \((z < z^*)\) not to pretend to be advisors:

\[ S(z, \tilde{z}) \leq R(z, z), \quad \text{for all } \tilde{z}. \]

With equality for the boundary type:

\[ S(z^*, z^*) = R(z^*, z^*) \]

(iii) **Ex post advice seeking.** Third, advice seeking must be ex post optimal when prescribed by the equilibrium. This requires that those with \(z < z^*\) must strictly prefer to seek advice, that is, \(V(z, z) > 0\), for all \(z < z^*\).

(iv) **Truthelling.** For originators and solvers to be willing to report truthfully their type we require that:

\[
\begin{align*}
V(z, z) &= \max_{\tilde{z}} V(z, \tilde{z}) \\
S(z, z) &= \max_{\tilde{z}} S(z, \tilde{z})
\end{align*}
\]

**Equilibrium Construction** To construct the equilibrium, we will first construct the marginal conditions for truthelling. We start by considering the problem of the originator who draws problem \(z\). For him to report truthfully his type we need that:

\[ V(z, z) = \max_{\tilde{z}} V(z, \tilde{z}), \]

that is, \(\forall z \in [0, z^*]::\)

\[
\begin{align*}
\left( \frac{\partial V(z, \tilde{z})}{\partial \tilde{z}} \right)_{\tilde{z} = z} &= 0 \\
-w'_e - \alpha_e \frac{m(\tilde{z}) - z}{1 - z} + (1 - \alpha_e) \frac{m'(\tilde{z})}{1 - z} &= 0
\end{align*}
\]

\(^8\)Since we require truthelling to be optimal conditional on choosing the right occupation we assume without loss that agents would be truthful if they choose the right occupation.
thus, in equilibrium ($\tilde{z} = z$)

$$-w'_z - \alpha'_z \frac{m(z) - z}{1 - z} + (1 - \alpha_z) \frac{m'(z)}{1 - z} = 0$$

or, since $m'(z) = h(1 - z)$:

$$-w'_z - \alpha'_z \frac{m(z) - z}{1 - z} + (1 - \alpha_z) h = 0 \quad (12)$$

While for a problem solver to report his type:

$$S(z, z) = \max_{\tilde{z}} S(z, \tilde{z})$$

that is, $\forall z \in [z_2, 1]$

$$\left(\frac{\partial S(z, \tilde{z})}{\partial \tilde{z}}\right)|_{\tilde{z} = z} = 0$$

$$\frac{1}{h} \left( w'_{m^{-1}(\tilde{z})} + \alpha'_{m^{-1}(\tilde{z})} \frac{\tilde{z} - m^{-1}(\tilde{z})}{1 - m^{-1}(\tilde{z})} \right) \frac{dm^{-1}(\tilde{z})}{d\tilde{z}} = 0$$

thus, in equilibrium ($\tilde{z} = z$)

$$w'_{m^{-1}(z)} + \alpha'_{m^{-1}(z)} \frac{z - m^{-1}(z)}{1 - m^{-1}(z)} + \alpha_{m^{-1}(z)} \frac{-(1 - z)}{(1 - m^{-1}(z))^2} = 0$$

or equivalently, in terms of worker skill, rather than manager skill we have:

$$w'_z + \alpha'_z \frac{m(z) - z}{1 - z} + \alpha_z \frac{-(1 - m(z))}{(1 - z)^2} = 0 \quad (13)$$

Adding equation (13) to (12) we have:

$$0 = (1 - \alpha_z) h + \alpha_z \frac{-(1 - m(z))}{(1 - z)^2}$$

which we can solve for $\alpha_z$

$$\alpha_z = \frac{h(1 - z)^2}{h(1 - z)^2 + (1 - m(z))}$$

This is a closed form solution for the $\alpha_z$ function—everything is known. Specifically, $\alpha_0 > 0$, and (since the denominator of $\alpha_z$ is larger than the numerator and it grows slower—recall that $m'(z) > 0$) $\alpha'_z > 0$. Moreover, $\alpha_z$ is:

---

9 The second order condition is:

$$-w'_z - \alpha'_z \frac{m'(z) - z}{1 - z} - 2\alpha'_z \frac{m'(z)}{1 - z} + (1 - \alpha_z) \frac{m'(z)}{(1 - z)^2} \leq 0.$$
\[
\alpha_{z^*} = \frac{h(1 - z^*)^2}{h(1 - z^*)^2 + (1 - m(z^*))} = 1
\]

thus the best expert is the full residual claimant of the output, and \( V(z^*, z^*) = -w_{z^*} \). Note that occupational choice \((ii)\) requires that \( w_{z^*} \leq 0 \) : the best originator is not getting any share, \( 1 - \alpha_{z^*} = 0 \), and he is passing the problem up, so he cannot be paying for advice.

The fact that \( \alpha'_z > 0 \) is intuitive since higher originators want to keep a lower share of the output and higher experts are willing to accept a larger fraction of variable compensation. Although one might be tempted to think that the fixed payments must be strictly decreasing to make up for the increasing variable share this is not the case.

Substitute \( \alpha_z \) in (12) to solve for \( w'_z \):

\[
w'_z = (1 - \alpha_z) h - \alpha'_z \frac{m(z) - z}{1 - z}
\]  

(14)

Since \( \alpha'_z > 0 \), and \( \alpha_{z^*} = 1 \), indeed \( w'_{z^*} < 0 \) but for \( z = 0 \) we have \( w'_0 = h - \alpha'_z z^* > 0 \). The non-monotonicity of the fixed fees arises from the fact that the matching function introduces an asymmetry in the local incentives to deviate. This asymmetry shows up in the difference in the last terms of 12 and \ref{eq:12} \cite{10}. In particular, consider the incentives for a type \( z = 0 \) from pretending to be a slightly higher type. Because the slope of the matching function is high close to zero, pretending to be slightly higher leads to a much better match for the originator. This incentive to exaggerate is partly offset by having \( \alpha' > 0 \) but note that the value of \( \alpha' \) cannot be chosen arbitrarily since it must simultaneously provide incentives for \( z^* \) consultants not to want to pretend to be of type \( z^* + \varepsilon \). Since the cost of exaggerating its type for an expert comes from receiving a harder match and the matching function is steep at zero, the \( \alpha' \) needed to satisfy the experts IC is lower than that necessary to satisfy the originators incentives. It is therefore necessary to have \( w'_0 > 0 \) to be able to satisfy both IC simultaneously.

From the fundamental theorem of calculus, given \( w_0 \) we can obtain the whole \( w_z \) schedule,

\[
w_z(w_0) = w_0 + \int_0^z \left( (1 - \alpha_t) h - \alpha'_t \frac{m(t) - t}{1 - t} \right) dt
\]  

(15)

\[
w_z(w_0) = w_0 + A(z)
\]  

(16)

The integral is involved and cannot be obtained in closed form. Yet, we can fully characterize this function for a given \( w_0 \). Since there are no independents for a given \( h \) supply and demand uniquely determine \( z^* \) (from \( Z_2(z^*) = z^* \)). Given \( z^* \), the matching function is uniquely pinned down (see equation 11) and so is \( \alpha_z \). The only object left to solve for is \( w_0 \) in (15). That is, the condition that

\footnote{This also explains why the contingent schedule on its own is not sufficient to provide incentives to both type of agents.}
the marginal originator \( z^* \) is indifferent between being an originator and becoming a consultant pins down \( w_0 \):

\[
S(z^*, z^*) = R(z^*, z^*)
\]

\[
\frac{1}{h}(w_0 + \alpha_0 z^*) = z^* (1 - z^*) w_2 \left( w_0, z^* \right), \text{ that is:}
\]

\[
w_0 = \frac{z^*(1 - \frac{1}{h} \alpha_0) - A(z^*, z^*)(1 - z^*)}{\frac{1}{h} + 1 - z^*}
\]

This completes the equilibrium construction. To verify that this is in equilibrium, note first that conditions (i), and (iv) are met by construction. Thus we need to verify that ex post advise seeking (condition iii) is optimal, that is \( V(z, z) > 0 \) for all \( z < z^* \).

\[
V(z, z) = -(w_0 + A(z, z^*)) + \left( \frac{(1 - (hz - \frac{1}{2} h z^2 + z^*) (hz - \frac{1}{2} h z^2 + z^* - z)}{(h(1 - z)^2 + (1 - (hz - \frac{1}{2} h z^2 + z^*)) (1 - z))} \right)
\]

\[
V(z, z) = -w_z (1 - \alpha_z) \frac{m(z; z^*) - z}{1 - z} + (1 - \alpha_z) \frac{(1 - z) (m'(z; z^*) - 1) - (m(z; z^*) - z)}{(1 - z)^2}
\]

and using the first order condition for truth telling:

\[
\frac{\partial V(z, z)}{\partial z} = -w'_z (1 - \alpha_z) \frac{m(z; z^*) - z}{1 - z} + (1 - \alpha_z) \frac{(1 - z) (m'(z; z^*) - 1) - (m(z; z^*) - z)}{(1 - z)^2}
\]

\[
\frac{\partial V(z, z)}{\partial z} = - (1 - \alpha_z) \frac{(1 - z) (h(1 - z) - 1) - (m(z; z^*) - z)}{(1 - z)^2} = \frac{(1 - \alpha_z) \left( (1 - z) (h(1 - z) - 1) - (m - z) - (1 - z)^2 \right)}{(1 - z)^2}
\]

which is negative if \( h - m + 4z - 2h z - z^2 + h z^2 - 2 < 0 \). Replacing \( m \) in and simplifying:

\[
h + 4z - 3h z - z^2 + \frac{3}{2} h z^2 + \frac{1}{h} \left( \sqrt{h^2 + 1} - 1 \right) - 3 < 0
\]

which is easy to verify to be true. Now we need to show that it is indeed always positive. Since \( \alpha_{z^*} = 1 \), this requires that \( w_{z^*} < 0 \); one can verify this is indeed the case.

Finally, we need to verify that occupational choice is optimal, that is condition (ii) holds. In fact, as we show in the Appendix, one can verify that this is indeed the case for \( h < h^* = .85 \); while for \( h > h^* \) there exist some advisors that prefer to exit the market.

**Proposition 8** There exists an \( h^* \) such that for \( h < h^* \) there exists a separating equilibrium with the properties that:

1. The optimal contract is characterized by an increasing share schedule \( \alpha_z \) and a fixed payment
$w_z$ which is a concave, single peaked function of $z$.

2. The share schedule $\alpha_z$ is increasing in $z$ for all $z$. The highest advisor is the residual claimant to the entire value of the problem ($\alpha_{z^*} = 1$).

3. An increase in communication costs $h$ increase the fixed price of advice (shift $w_z$ up) and reduce the share of the solution retained by advisors ($\alpha_z$ goes down).

Intuitively, truthtelling is attained through both the fixed and variable portions of the contract and the quality of the match. In fact, since $w_z$ is non-monotonic, there exist $z < z'$ such that $w_z = w_{z'}$ and yet truthtelling is attained even though $\alpha_z < \alpha_{z'}$. The high type $z'$ would seem to prefer the contract for $z$, since it costs the same fixed payment to get advice and a lower share must be offered (and thus a higher kept). However, at that price and share the advice received is worse, since $m(z) < m(z')$; this ensures truth-telling. Conversely, the low type $z$ does not prefer the better advise since that requires offering a higher share, and the problem is sufficiently easy that it can be solved by the worse advisor with relatively high probability.

Finally, we can compare this equilibrium with the first best. It is clear that, as long as the equilibrium exists, trade is weakly larger than in the first best. Moreover, matching is worse. The asymmetric information helps the ‘worse’ agents, who in this economy are the intermediate ones—those are the least skilled advisors and the most skilled (thus less value to add) originators of problems.

**Proposition 9 (Inefficiencies)** Compared with the first best, the competitive equilibrium with double sided asymmetric information is characterized as follows:

1. There is (weakly) too much advice: as long as the market is functioning, too many agents whose problems are hard are seeking advice, and too many advisors with little knowledge are giving advice.

2. Originators are getting weakly worse advice than in the first best. Advisors are getting weakly harder problems.

3. The best problem solvers are worse off and the worst originators are worse off than in the first best (those in the extremes of the skill distribution), while the worst problem solvers and best originators are better off.

**Proof.** 1 follows immediately from the previous proposition: as long as the market is functioning there are no independents, whereas the first best has independents for $h > .75$.

2. Also follows immediately. $h > .75$ there are low quality advisors in the market, who would be independent in the first best. Those are dealing with the easiest problems, similarly, there are high skilled originators (with difficult problems) also participating in the market.
3. Clearly, those who were independents in the first best (middle types) are now in the market, thus must be better off (they always could originate their own problems and keep them). To see that the highest type is worse off when the independents were around, note that in the first best he is attaining a value of $1/h$ as long as there are independents- he pays 0 (to the last originator, who is indifferent between seeking and not seeking advice) and solves the probability $(1 - z_1)/(1 - z_1)$; but in the asymmetric information equilibrium there are no independents, and thus he must share some rents- pay for the problem- he still solves it for sure, but now at a cost. Similarly, the lowest originator is obtaining the rents of the problem, that is $w_0 = 1/h(z_2 - 0)/(1 - 0) - z_2 = z_2(1 - h)/h$. If $z_2$ decreases (as must happen when there are no independents), clearly this rents decrease; moreover, if there are no independents, there are rents going towards the advisor: $1/h(z^* - 0)/(1 - 0) - w_{z^*}$:

$$\frac{1}{h}z_2 - z_2 > \frac{1}{h}z^* - z^* > \frac{1}{h}z^* - w_{z^*}$$

Thus clearly the best and worst agents are worse off than in the first best, and by continuity there is an interval of the best and worst agents some other agents are worse off as well. [note only shown for no independents case. ■

IV Certification

As we saw in Section III when output is unverifiable and ownership cannot be transferred there does not exist a competitive equilibrium with trade. Recall that the difficulty of sustaining an equilibrium came from the low types pretending to be experts. A way a social planner could address this problem is by regulating entry into the expert services market. In fact, in many professions there exists some sort of certification with the purpose of signalling to potential consumers that the professional they are hiring has a level of expertise above some threshold. This can be rationalized if the person hiring the services of the expert cannot observe his ability level directly.

For example, in its description of the bar exam, the American Bar Association says the purpose of the license is to establish competence, through a showing that "the applicant holds an acceptable educational credential (with some exceptions, a J.D. degree) from a law school that meets educational standards, and by achieving a passing score on the bar examination.(...) [The exam's] purpose is to protect the public."

With this in mind, consider a benevolent planner that has the ability to set an exam of difficulty level $c$ which allows agents with types higher than $c$ to become certified experts. The question we study next is what is the optimal level of certification $c^{*}$?

For a given level of certification $c$, those above that level may choose to be consultants or independents, those below are originators who may or may not choose to seek advice for their unsolved problems. Note first that if an originator of type $z$ is willing to pay a fee to seek advice then any
other originator of type \( z' < z \) will also be willing to pay such a fee.\(^{11}\) Thus there exists a \( z_1 \) such that all agents \( z < z_1 \leq c \) seek advice. Second there may or may not exist an interval \([z_1, c]\) of originators who do not seek advice. Third, without loss we can rule out the case in which there would be independents in an interval \([c, z_i]\), since the cutoff \( c \) could then be raised at no cost. Fourth, note that if an agent \( z > c \) prefers to be an independent rather than a consultant, then all agents \( z' > z \) would also choose to be independent.\(^{12}\) In principle there can be a set of high types \([\bar{z}, 1]\) that do not participate in the market.

A competitive equilibrium with certification can be defined as:

**Definition 7 (CE with Certification)** A Competitive Equilibrium with Certification consists of:

A certification level \( c \), a fee for consulting services \( w \) and cutoff \( z_1 \) and \( \bar{z} \) such that:

i) Entry to the expert sector is optimal: \( F(\bar{z}) \leq \frac{w}{h} \) with equality if \( \bar{z} < 1 \).

ii) Seeking expert advice is optimal for \( z \leq z_1 \): \( w \leq \Pr(q < m|q > z, m \sim U(c, \bar{z})) \) with equality if \( z_1 < c \).

iii) Market Clears: \( \int_0^{z_1} (1 - F(z)) dz = \frac{\bar{z} - c}{h} \)

iv) Feasibility: \( z_1 \leq c \leq \bar{z} \leq 1 \)

The total output in the CE with Certification is:

\[
Y = \int_0^{z_1} \left( F(z) + \frac{1 - F(z)}{\Pr(q < m|q > z, m \sim U(c, \bar{z}))} \right) dz + \int_{z_1}^{c} F(z) dz + \int_{\bar{z}}^{1} F(z) dz
\]

The planner can pick from different competitive equilibria by changing the certification level to maximize total output. Alternatively, subject to the proper constraints, we can think of the planner as choosing the size and level of expertise in the market as determined by the triple \( \{z_1, c, \bar{z}\} \). Where \([0, z_1]\) determines the size of the market for advice and \([c, \bar{z}]\) determines the average quality.

**Conjecture 1** For all \( h \in (0, 1) \) the Planner will always optimally restrict entry into the consulting sector by setting \( c \) strictly above the first best cutoff type \( z_2 \).

Recall that in the first best it is always the case that the best agents participate in the consulting market i.e. \( \bar{z} = 1 \). However, with information constraints, the consulting fees are the same for all experts which particularly hurts the highest types. Hence, these agents might now prefer to abandon the market. By increasing the certification level the planner can increase the earnings of these agents to the point they again want to participate in the market. In a sense the planner faces a quantity versus quality trade-off because by increasing \( c \) he also excludes some agents from the

---

\(^{11}\)Since the problems he seeks advice on are in expectation easier the lower the originator’s type, the payoff of receiving advice are higher in expectation.

\(^{12}\)This follows because while the income of an expert is not a function of his type, the income of an independent is increasing in his type.
consulting market. The distribution of problems determines the extent of this trade-off. If there are very few hard problems then there is little gain in bringing in the highest types and a lot lost by increasing \( c \). Hence, in general it might be optimal to leave the smartest agents out of the market but, in the uniform case it can be shown that this is never the case, and thus the problem can be written in a simplified way as:

\[
\max_{c, z_1, z} \int_0^{z_1} (F(z) + (1 - F(z)) \Pr(q < m|q > z, m \sim U(c, 1))) \, dz + \int_c^c F(z) \, dz
\]

where:

\[
\Pr(q < m|q > z, m \sim U(c, 1)) = \int_c^1 \frac{F(m) - F(z)}{1 - F(z) (1 - c)} \, dm
\]

hence:

\[
\max_{c, z_1} \int_c^1 \frac{F(m)}{1 - c} \, dm + \int_c^c F(z) \, dz + \int_{z_1}^c z \, dz + z_1 \frac{(1 + c)}{2}
\]

Finally, since \( F(q) \) is uniform we can rewrite the problem simply as

\[
\max_{c, z_1} \int_{z_1}^c z \, dz + z_1 \frac{(1 + c)}{2}
\]

For \( c, z_1, \bar{z} = 1 \) to be a competitive equilibrium conditions \((i)\) to \((iv)\) must be met.

Occupational choices (conditions \((i)\) and \((ii)\)). First, since all consultants earn the same fees—the best consultant must prefer to accept fees \( w \) and work in consulting earning \( w/h \) rather than originate and solve his own problems with probability 1 and earn 1, that is:

\[
1 \leq \frac{w}{h}
\]

Second, the marginal originator \( z_1 \) must indeed prefer to pay a consulting fee rather than drop a problem he cannot solve. The cost of hiring a consultant is the consulting fee \( w \); the benefit is the conditional probability that a consultant of average quality, \( \frac{c+1}{2} \), can solve the problem, given that \( z_1 \) could not.

\[
w \leq \frac{\frac{c+1}{2} - z_1}{1 - z_1}
\]

These two conditions imply a necessary condition for equilibrium:

\[
h \leq \frac{-h(z_1 - \frac{c+1}{2} - z_1^2) + 1}{1 - z_1} - z_1
\]

\((17)\)
Market clearing \((iii)\) for the uniform case simplifies to:

\[
\frac{1 - c}{h} = z_1 - \frac{z_1^2}{2}
\]

or equivalently:

\[
c = -h \left( z_1 - \frac{1}{h} - \frac{1}{2} z_1^2 \right)
\]  \hspace{1cm} (18)

Finally, feasibility \((iv)\) requires that:

\[
z_1 \leq c
\]

The last two conditions together imply:

\[
z_1 \leq -h \left( z_1 - \frac{1}{h} - \frac{1}{2} z_1^2 \right)
\]  \hspace{1cm} (19)

It can be shown that for \(h < .5\), the inequality that may bind is \(z_1 < c\), or \((19)\) inequality \((17)\) is met if \(z_1 \leq c\). Intuitively, when providing advice is a very productive activity high types are happy to participate in the market. At the same time, since \(h\) is low the market clearing condition implies that only a small measure of agents are needed in the expert market hence the average quality of the services provided is high and originators want to bring their problems to the market. Conversely, for \(h > .5\), inequality \((17)\) may bind and inequality \((19)\) is never binding.

**Proposition 10 (Too Little Advice)** The second best credentialing optimum is a pooling equilibrium characterized as follows:

1. For \(h < .41667\), there are no independent originators \(z_1 = c\). There is a continuum of fees \(w \in [h, 1/2]\) compatible with the equilibrium.

2. For \(0.41667 < h < 0.692\): there is an interior equilibrium with independents; the fees are pinned down by the indifference condition of originators who do not seek advice.

3. For \(0.692 < h < 1\) the fees are such that (1) the marginal originator is indifferent between having the problem solved or not; and (2) the marginal consultant is indifferent between being a consultant or an independent. The number of consultants in this equilibrium is distorted downwards.

When \(h\) is low the certification is set at the same level it would be set absent any informational considerations. The only inefficiency arises because, instead of assortative, the matching is now random. As \(h\) increases, providing advice becomes less efficient, the fees for the consulting increase and high originators lose interest in seeking expert help in the market. This leads to an equilibrium with independents. Entry is being restricted relative to first best because high originators prefer
to deal with their own problems without participating in the market for advice. Finally, when \( h > 0.692 \) experts can solve very few problems and hence unless fees are increased they have an incentive to leave the market. In order to have originators willing to pay higher fees, the average quality of the expert sector must be increased and this is done by further restricting entry.

V Empirical Implications and Conclusions

A One-sided asymmetric Information in Problem Solving Markets: Consulting markets

As we show in Section B, when information about the quality of those giving advice can be easily obtained (maybe through reputation or through well functioning certification mechanisms) contracts should take the form of consulting contracts: expertise is provided in exchange for a fee; those buying the advice can easily internalize the difficulty of their own problem, and the consulting fee internalizes properly the match between the expected difficulty of the problem and the skill of the consultant required. This is consistent with the use of consulting by firms in many contexts, where essentially the consultant names a price and a quality pair and the client sorts among firms.\(^{13}\)

On the other hand, when opportunities are transferable and the quality of experts who would be appropriate for a given opportunity is more difficult to observe, we expect referral contracts to be preferred. In this context, originators post their opportunities in exchange for a fee and experts bid for them. The market price for these opportunities will be such that, again, experts will sort themselves so that the best expert will end up with the a priori more difficult opportunities. Such markets are observed in biotech, for example, where firms which have discovered molecules and want to take them to market try to find the right company to do this by selling their IP, the profitable opportunity they generated; they post the opportunity and idea, and the pharma companies sort themselves among opportunities.

B Markets for advice with two-sided asymmetric information

In Section 3 we studied the design of expert markets when asymmetric information is relevant on both sides – the originating and the problem solving side. We argue, first, that whether a solution exists depends on the extent to which output is verifiable; if it is not, absent certification the market will disappear. If it is, then we find that (1) the optimal contract involves a fixed payment and an equity stake for both problem solvers and originators; that (2) the equity share increases with the quality of the problem solving required; (3) that income distribution in the market is tighter than in the first best, as rents are capture by those agents in the middle of the distribution, either the worst originators (the most skilled ones) or as worst problem solvers (the least skilled ones among

\(^{13}\)Of course, there is an element of risk sharing in the hourly fee structure, but the total price of the project is actually basically known in advance with a high degree of certainty in this market.
them); and that (4) there is too much problem solving in equilibrium, as all problems are solved. We discuss next three instances of problem solving markets under asymmetric information.

**Online Expert Markets**  A range of companies have emerged to help companies get problem solving advice. The pioneer in this industry is Innocentive, other rivals are Innovation Exchange, Fellowforce, NineSigma, yet2.com, and YourEncore.\(^{14}\) The market has two sides, those who post problems for which no solution is yet known, called the ‘seekers’ by Innocentive and those who attempt to provide a solution who are called ‘solvers’ in the site. Seekers post ‘challenges’ which are unsolved problems. Like in our case, there is asymmetric information both about how difficult the challenge will eventually prove and about the skill of those attempting a solution.

The leading site at the moment is Innocentive.com. The site had, as of May 2008, 145000 solvers registered, who have submitted to date 7,011 solutions; 620 challenges have been posted, with a total award of $16m, of which 215 have been solved with $3m paid out.\(^{15}\)

This generation of sites operate along the lines of a tournament model; a prize is posted, and it is awarded to the agent who solved the challenge. The system has an important inefficiency— the effort of those who do not win the challenge is wasted. Moreover, this inefficiency is compounded strategically, as participants try to figure out which challenges will attract just the right number of solvers to ensure an adequate probability of winning.\(^{16}\) Of course, it is hard to know a priori how hard and how attractive a challenge will be, but the system as set up has the seed of its own destruction. If it becomes too popular, the probability of being the chosen solution collapses, and those with a substantially higher opportunity cost of time— presumably the best solvers— drop out of it.

Our analysis suggest that the system should be replaced by one in which a restricted number of solvers, potentially grouped into a team, are given an opportunity to solve the problem in exchange for a fee and a share in the output that may result, where the share should be higher the harder the problem. This will, unlike a tournament, attract the right level of talent. Absent an arrangement along these lines, these sites are likely to remain small and out of the mainstream, attracting only the non-mainstream solvers.

**Rent sharing and Referrals in the Law**  Like in our model, lawyers generally pass on clients to one another in exchange for a referral fee. This is particularly the case in litigation, where these payments take the form, as in the contracts we describe, of referral shares. While such compensation

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\(^{15}\) This information is from the InnoCentive.com (A) HBS case 9-6098-170, by Karim R. Lakharni, dated June 10, 2008.

\(^{16}\) A top solver in Innocentive, David H. Tracy declared “I’m good enough mathematician (barely) to know better than to play the lottery... If I thought that a given Challenge would attract say 100 strong solutions— solutions likely to be roughly as wonderful as mine— then I might choose not to invest the time needed to create and submit a solution with just 1% probability of winning.”
arrangements involve clearly team production and moral hazard issues as well (see Garicano and Santos, 2004), there is also a sorting element along the lines of our analysis. Thus, we expect better lawyers to receive larger shares of output, and to be matched with harder problems. Empirically we should find referral shares increasing with the quality of the claim or of the lawyer.\textsuperscript{17}

**Venture Capital: Sorting and Contracting** Venture capital markets have very similar features to the ones in our second best contracts. Those who originated a business idea must find experts that help them take them to market. Venture capital contracts generally will involve cash transfers and equity stakes— that is a share in the profits if the idea is successful.

Our model has clear implications for this type of markets. First, as pointed out above, there should be positive sorting between the quality of the deal and the quality of the venture capitalist; only a good venture capitalist is able to add sufficient value to a good entrepreneur. A direct test of this is Sorensen (2007), who finds that more experienced venture capitalists make more successful investments and invest in ‘better’ companies— late stage and biotechnology companies. Second, this positive sorting should covary with increasing revenue shares for the venture capitalist— the higher the unobserved quality of the entrepreneur (and of the venture capitalist) the higher should be the revenue share accruing to the venture capitalist, as the better entrepreneur signals good quality by offering a large residual share. Kaplan and Stronberg (2002) come closest to being able to test this, as they have contracting data on VC contracts; however, their regressions do not test for these sorting and share effects.

**C Licensing in Professional Services**

Finally, Section IV applied our analysis to the study of certification in professional service markets. A long running discussion in the licensing literature has sought to clarify the extent to which professional licensing is a consequence of expert capture (along Friedman’s and Stigler’s views) or regulation under asymmetric information. Some empirical papers have found that, contrary to what an optimality view would contend, it is in fact the case that licensing restrictions reflect capture by the professions.

Our analysis suggests that licensing must be restrictive, and that efforts to keep the top experts in the pool may indeed lead to shrinking the pool beyond the second best without occupational choice. Essentially, when providing advice is costly and separating good from bad advisors is hard, two types of inefficiencies take place: first, the top originators are less interested in seeking advice, at it is unlikely to be effective; second experts want to leave the market, as the pay received as an average expert is too low. Attaining second best efficiency requires restricting further the quality of experts, leaving outside some experts who would be experts in a first best world.

\textsuperscript{17}A test along these lines was conducted in small sample by Stephen Spurr (1988). However, rather than presenting the regression of share on either claim value or quality of lawyer he includes both in the only specification he studies and finds them insignificant.
REFERENCES


VI  Appendix

**Lemma (1) Assortative Matching.** We show that $s < s'$ for $z > z'$ cannot be optimal. Conditional on being consulted on one problem each, the expected number of solved problems is:

$$\frac{F(s)}{1 - F(z)} + \frac{F(s')}{1 - F(z')}$$

If instead we reverted the matching so that type $m(z)$ tries to solve type $z'$ problems and vice-versa the number of solved problems would be:

$$\frac{F(s')}{1 - F(z)} + \frac{F(s)}{1 - F(z')}$$
We show that the second arrangement is more productive if $s < s'$ for $z > z'$:

\[
\begin{align*}
F (s') & > F (s) \\
F (s') (F (z') - F (z)) & < F (s) (F (z') - F (z)) \\
F (s') F (z') - F (z) F (s') & < F (s) F (z') - F (z) F (s) \\
F (s') F (z') + F (z) F (s) & < F (s) F (z') + F (z) F (s') \\
F (s') (1 - F (z')) + (1 - F (z)) F (s) & > F (s) (1 - F (z')) + (1 - F (z)) F (s') \\
\frac{F (s')}{1 - F (z)} + \frac{F (s)}{1 - F (z')} & > \frac{F (s)}{1 - F (z)} + \frac{F (s')}{1 - F (z')}
\end{align*}
\]

**Lemma (2) Independents are Smart.** If $z' > z$ could not solve a problem it means that the problem is harder to solve than the unsolved problem by type $z$. Hence, it is more likely that type $m (z')$ will solve problem $z$ than problem $z'$ therefore the planner would be better off by leaving $z'$ unmatched and matching $z$. This implies that no originator can be smarter than an independent.

**Lemma (3) Experts are Smarter.** Consider two agents and independent $z_I \in I$ and a consultant with type $m (z)$. Assume in search of a contradiction that $z_I > m (z)$ is optimal. The joint output of these two types is:

\[
F (z_I) + \frac{F (m (z))}{h F (z)}
\]

Since $\frac{1}{h} > 1$, if $z_I > m (z)$ then:

\[
F (m (z)) + \frac{F (z_I)}{h F (z)} > F (z_I) + \frac{F (m (z))}{h F (z)}
\]

Therefore the planner could improve by having $z_I$ and $m (z)$ switch their roles. Hence consultants must always be smarter than independents.

**Proposition 11 (2)** The number of independent agents is increasing in $h$. If $h > h'$ then $I' \subseteq I$.

**Proof.** Follows from taking the derivative of the FOC with respect to $h$. Details to be added.

**Proposition 12** The competitive equilibrium allocation exists and is unique. It may be implemented equivalently through a referral or a consulting market. It attains the first best.

We shall show that the competitive equilibrium is unique and attains the first best. To do this we abstract first from the actual implementation for now proceed and proceed through a series of three lemmas which follow below. We then show that the earnings and allocation in the referral and consulting formulations match the ones in the general derivation we follow.
Lemma 7 The competitive equilibrium must display positive assortative matching.

Proof. To see this consider the production function of a firm that hires solvers and originators of skill \( z_s \) and \( z_o \). This firm’s production function will be:

\[
\pi(z_s, z_o) = F(z_s)n - w_o n - w_s
\]

Subject to the time constraint of the problem solver, \( h(1 - F(z_o)n) = 1 \). That is the profit function of this firm is:

\[
\pi(z_s, z_o) = \frac{F(z_s) - w_o}{h(1 - F(z_o))} - w_s
\]

It is clear this production function displays increasing differences (since \( \frac{\partial^2 y}{\partial z_o z_s} > 0 \) where \( y = F(z_s)n \)) and thus positive sorting must hold in equilibrium.

Lemma 8 (Market Clearing) Equality of supply and demand means that the competitive equilibrium is pinned down up to the two cutoffs \( z_1 \) and \( z_2 \). Moreover, for each \( z_1 \) there exists a unique \( z_2, z_2^{sd}(z_1) \) such that supply equals demand. Finally, \( z_2^{sd} < 0 \).

Proof. Suppose first that some agents are unmatched—there are independent originators. Supply equals demand pointwise implies: \( m'(z) = (1 - F(z))h \). With \( m(0) = z_2 \), we can write the matching function as \( m(z; z_2) \). Then for a given \( z_2 \) and

\[
m(z_1; z_2) = 1
\]

implies that the match is entirely pinned down up to one constant \( z_1 \). Trivially, \( m(z_1; z_2) = 1 \) implies a function \( z_2^{sd}(z_1) \) with \( z_2^{sd} < 0 \) (intuitively, if there are more supply of problems, you need a larger supply of problem solvers).

Lemma 9 (Uniqueness) There always exists a competitive equilibrium for \( h < 1 \). This equilibrium is unique.

Proof. The proof is by construction. We move along the \( z_2^{sd}(z_1) \) curve until either \( z_2 = z_1 \) or \( z_2 = w_c(z_2) \).

Consider a profit maximizing firm that hires teams of originators \( z_0 \) and problem solvers \( z_s \). Given that each problem solver can solve \( 1/h \) problems per unit of time, and that an originator only needs help with probability \( (1 - F(z_o)) \), the firm will need a measure \( n = 1/(h(1 - F(z_o))) \) of originators per problem solver, so that earnings are given by:

\[
\pi(z_s, z_o) = F(z_s)n - w_o n - w_s
\]
by the 0 profit condition these can be through of equivalently as the measure of consultants hires the originators:

\[ w_s(z_s, z_o) = F(z_s)n - w_on = \frac{F(z_s) - w_o}{h(1 - F(z_o))} \]

For the choice of originators of quality \( z_o \) to be an optimum, it must be the case that the wages are such that the choice of \( z_o \) is optimum:

\[ w_s(z_s, z_o) = \max_{z_o} \frac{F(z_s) - w_o(z_o)}{h(1 - F(z_o))} \]

From here, using the first order condition and then the envelope we can obtain the slope of the earnings curve along the equilibrium:

\[
\begin{align*}
\frac{\delta w_o(z)}{\delta z} &= \frac{f(z)}{(1 - F(z))} \left( F(m(z)) - w_o(z) \right) < f(z) \left( \frac{F(m(z)) - F(z)}{1 - F(z)} \right) < f(z) \\
\frac{\delta w_s(z)}{\delta z} &= \frac{f(z)}{h(1 - F(m^{-1}(z)))} > f(z)
\end{align*}
\]

Where we are using the matching schedule definition \( z_s = m(z) \). The inequality in the first line uses the fact that, for originators who actually choose to be originators, earnings as originators are higher than earnings as self employed.

Now we move along \( z^{sd}_1(z_2) \). Since the top consultant matches with originator \( z_1 \) and \( w(z_1) = F(z_1) \) for optimal occupational choice, top consultants earnings are fixed at \( \frac{1 - F(\varepsilon)}{h(1 - F(\varepsilon))} = \frac{1}{h} > 1 \) as long as \( h < 1 \), and as long as the equilibrium is interior (there are originators). The earnings schedule of consultants thus starts at \( w_o(1) = 1/h \) and decreases with slope \( \frac{f(z)}{1 - F(m^{-1}(z;z_2))} \). Specifically, since increasing \( z_1 \) raises the value of the match of every problem solver, this means that the rate of decrease of earnings as we reduce \( z \) is larger the higher \( z_1 \) is. Thus \( \frac{d}{dz_1} \left( \frac{f(z)}{1 - F(m^{-1}(z;z_2))} \right) > 0 \). Start from \( z_2 = 1, z_1 = 0 \) We know this is a market clearing pair (that is \( z^{sd}(1) = 0 \)), since there is no supply or demand of problems. The worst workers earn \( F[0] = 0 \) and the best ones earn \( F[1] = 1 \). Now consider a deviation along the market clearing condition so that \( z_1 = \varepsilon \) and \( 1 = z^{sd}_2(z_1) \). Now the value of the match is \( \frac{1 - F(\varepsilon)}{h(1 - F(\varepsilon))} = \frac{1}{h} > 1 \) as long as \( h < 1 \). Managers clearly will chose to hire workers \( \varepsilon \), pay them \( z_1 = \varepsilon \) and earn themselves \( 1/h > 1 \). However, this is not an equilibrium, as the agents at \( z_2 = 1 \) strictly prefer being problem solvers than independents (the earnings function is discontinuous at \( z_2 = 1 \)). Raise now \( z_1 \) to \( z^*_1 = 2\varepsilon \). Now earnings of top solvers \( z = 1 \) are still \( 1/h \). Construct the earnings function of consultants by using \( \frac{f(1)}{1 - F(2\varepsilon))} \). The earnings of \( z^*_2 = z_2(2\varepsilon) \) are either still \( w(z^*_2) > z_2 \) or \( w(z^*_2) = z_2 \). In the second case, we have a competitive equilibrium and stop. In the first case, we go back and increase \( z_1 \) again by \( \varepsilon \). Now the slope of the earnings function at 1 is stepper at every point, \( \frac{f(1)}{1 - F(3\varepsilon))} > \frac{f(1)}{1 - F(2\varepsilon))} \) etc. Since \( \frac{f(z)}{1 - F(m^{-1}(z_1;z_2))} > f(z) \), and \( z^*_2 < z_2 \) the distance \( w(z_2) - z_2 \) is unambiguously reduced with each step. We can continue taking
these steps till $z_1 = z_2$. If at any point $w(z_2) = z_2$, we have an equilibrium, since $w(z_1) = z_1$, market clears, and matches are optimal (agents cannot gain by deviating since, by construction, the slope is always equal to the marginal contribution. Moreover, since $\frac{\delta w(z)}{\delta z} < f(z)$, if worker $z_1$ is indifferent between being a worker or an originator, all workers with $z < z_1$ strictly prefer to be workers. If instead at this point it is still the case that $w(z_2) > z_2$, then we have no independents, and we can obtain the cutoff simply from the market clearing condition, $z_1 = z_2 = z^*$, where $m(z^*; z^*) = 1$. 

**Lemma 10** The unique competitive equilibrium is Pareto Optimal.

**Proof.** (A) Suppose first that the CE has ‘more trade’ that is there are problems that had no value in the first best that are now traded. Note that this can only happen if in the first best first best $z_1 < z_2$. If $z_1 = z_2$ demand=supply would prevent the possibility of having more trade than in the first best.

Intuitively, too much trade relative to first best implies all agents are matched with more difficult problems than in the competitive equilibrium. Therefore, they are earning less and thus the marginal agent prefers to be in fact (contrary to the statement) independent.

That is, we show that there cannot be ‘too much trade’. Letting $w_s(z; z_2)$ denote the expected earnings of a consultant of type $z$ given the cutoff is $z_2$ then: $z^{CE}_2 < z^{FB}_2 = F(z^{FB}_2) < w_s(z^{FB}_2; z^{CE}_2) < w_s(z^{FB}_2; z^{FB}_2) = F(z^{FB}_2)$ a contradiction. $F(z^{FB}_2) < w_s(z^{FB}_2; z^{CE}_2)$ follows from the fact $z^{CE}_2 < z^{FB}_2$ and $w_s(z^{CE}_2; z^{CE}_2) = F(z^{CE}_2)$ since $z^{CE}_2$ is the cutoff type. Similarly $w_s(z^{FB}_2; z^{FB}_2) = F(z^{FB}_2)$. Finally, to show $w_s(z^{FB}_2; z^{CE}_2) < w_s(z^{FB}_2; z^{FB}_2)$ first, note that $z^{CE}_2 < z^{FB}_2$ implies that $m^{-1}(z; z^{CE}_2) > m^{-1}(m(z; z^{FB}_2))$. Also, recall that $\frac{\delta w_s(z; z_2)}{\delta z} = \frac{f(z)}{m(1-F(m^{-1}(z; z_2)))}$ and thus $\frac{\delta w_s(z; z^{CE}_2)}{\delta z} > \frac{\delta w_s(z; z^{FB}_2)}{\delta z}$ for $z \geq z^{FB}_2$. Then since there are independents, $w_s(1; z^{CE}_2) = w_s(1; z^{FB}_2) = \frac{1}{h}$. Therefore, $w_s(z^{FB}_2; z^{CE}_2) < w_s(z^{FB}_2; z^{FB}_2)$ follows. If there were no independents in the competitive equilibrium it must be that $w_s(1; z^{CE}_2) < \frac{1}{h}$ and the result follows as well.

(B) Conversely, there cannot be ‘too little trade’. Intuitively, all problem solvers would be better off (getting easier problems) compared to the first best, leading again to a contradiction

That is, we show that there cannot be ‘too little trade’ $z^{CE}_2 > z^{FB}_2 = F(z^{CE}_2) = w_s(z^{CE}_2; z^{CE}_2) > w_s(z^{CE}_2; z^{FB}_2) = F(z^{CE}_2)$ a contradiction. To show that $z^{CE}_2 > z^{FB}_2 = F(z^{CE}_2) = w_s(z^{CE}_2; z^{CE}_2) > w_s(z^{CE}_2; z^{FB}_2)$ first, note that $z^{CE}_2 > z^{FB}_2$ implies that $m^{-1}(m(z; z^{CE}_2)) < m^{-1}(m(z; z^{FB}_2))$. Therefore, $\frac{\delta w_s(z; z^{CE}_2)}{\delta z} < \frac{\delta w_s(z; z^{FB}_2)}{\delta z}$ for $z \geq z^{CE}_2$. Since $w_s(1; z^{FB}_2) \leq w_s(1; z^{CE}_2) = \frac{1}{h}$ (with equality if $z^{FB}_2 = z^{FB}_1$), it is clear that $w_s(z^{CE}_2; z^{CE}_2) > w_s(z^{CE}_2; z^{FB}_2)$ follows. 

**Lemma 11** The referral market formulation yields the first best.

**Proof.** It suffices to show that the earnings functions in both alternative equilibrium are the same as the one in the competitive equilibrium we showed above up to the constants $z_1$ and $z_2$. The earnings that originators obtain from the match are $r(z_o)$; the slope $r'(z_0) = \frac{f(z_o)(1-F(z_o))}{(1-F(z_o))^2}$ as
prescribed. Second, the earnings of consultants are \( w(z_s) = \max_{z_o} \frac{1}{h} \left( \frac{F(z_s) - F(z_o)}{1 - F(z_o)} - r(z_o) \right) \); thus from optimality \( w'(z_s) = \frac{f(z_s)}{1 - F(z)} \), again as prescribed. ■

**Proposition (5).** If experts are paid a fixed fee \( \phi \) and a given type \( z \) chooses to become an expert then all types \( z' < z \) will choose to become experts as well. This follows from noting that for type \( z \):

\[
\frac{\phi}{h} \geq F(z) + (1 - F(z)) (\max \{0, \Pr(sol|z) - \phi\})
\]

where \( \Pr(sol|z) \) is the probability that the problem gets solved conditional on hiring an expert and the difficulty of the problem being above \( z \).

Furthermore, since type \( z \) could choose to not solve a problem of difficulty \( q < z \) it must also follow that he can pretend his ability level is \( \tilde{z} < z \) and therefore:

\[
\frac{\phi}{h} \geq F(\tilde{z}) + (1 - F(\tilde{z})) (\max \{0, \Pr(sol|\tilde{z}) - \phi\}) \quad \forall \tilde{z} < z
\]

but the RHS of the equation is exactly what any type \( \tilde{z} < z \) would get. Note that these agents can also get \( \frac{\phi}{h} \) since their type is not verifiable. Hence, they would all choose to become experts. As a result, there would be nobody interested in hiring an expert because experts would not be able to solve the problem. ■

**Proof of Lemma 5.** In search of a contradiction, suppose that \( m'(z) \) is negative. Suppose first that there are no independents. This would imply that the lowest problem solver \( z^* \) is meant to solve problems for the best originator, \( z^* \). Clearly, no problem posed is solved, \( z^* - \tilde{z}^* = 0 \) and hence there cannot be any trade between them. Second, suppose there are independents. Then type \( z_1 \), the highest originator must turn to \( z_2 \), the lowest problem solver, for help. The gains to \( z_1 \) from hiring \( z_2 \) must be 0, since he must be indifferent between getting help or not:

\[
0 = -w_{z_1} + (1 - \alpha_{z_1}) \frac{z_2 - z_1}{1 - z_1}
\]

while \( z_2 \) must be indifferent between becoming a problem solver or an independent:

\[
z_2 = \frac{1}{h} \left( w_{z_1} + \alpha_{z_1} \frac{z_2 - z_1}{1 - z_1} \right)
\]

This \( z_2(z_1) \) is generically different that the one required to satisfy market clearing. ■
Proof of Lemma 6. We show all the problems are solved in the separating equilibrium. ■

We first construct the equilibrium It must be that \( V(z_1, z_1) = 0 \). This implies \( w_{z_1} = 0 \). Thus, from equation (16):

\[
\begin{align*}
w_0 &= -A(z_1, z_1) \\
&= - \int_0^{z_1} \left( (1 - \alpha_t) h - \alpha_t \frac{m(t; z_1) - t}{1 - t} \right) dt
\end{align*}
\]

To solve for \( z_1 \), we use the occupational choice condition: agent \( z_2 \) must be indifferent between working as a consultant and working as an independent:

\[
Z_2(z_1) \left( \frac{h}{h + 1 - Z_2(z_1)} - h \right) = A(z_1, z_1)
\]

(20)

We have now solved for \( w_0 \) and \( z_1 \). Given this boundary values, the equilibrium is uniquely identified. To show that this candidate equilibrium is not, in fact, an equilibrium, we show that it violates conditions 2 and 3. we do it by showing that the total output of the match implied by this \( z_1 \) and \( w_0 \) is less than in autarchy for some agents. In particular, figure XX shows that for agents \( z_1^{*}/2 \) and \( m(z_1^{*}/2) \) autarchy always results in better matches than the separating equilibrium.